How To Study Maths Effectively?
IUCAA Lecture Programme for School Students

August 9, 2008

Dear students,

It is my great pleasure to interact with you. As a teacher, it is my main desire to help you realise, the great potential each one of you have. I have been training young students for several years. Every year number of my students are selected at the State and National level Olympiad examinations and also at IIT Entrance examination. Some of these students have won medals even at the International Mathematical Olympiad. Today I wish to share with you some of the effective methods of studying. (Mathematics in particular.)

Deceptive Success
Most of the students have been brain washed to believe that, their main aim is to score maximum marks in an examination. So, such students memorise the answers to number of questions and do get very high percentage of marks in the board type examinations. Do you know what happens to them subsequently? It is well known that the cut off level, for engineering admission (in a decent college) is above 90%. Last year about 16,000 students appeared for the First Year Engineering Examination. Only 4,000 managed to pass. Among those who passed, only 400 secured first class.

Reasons of failure
Let us look at the nature of the Xth board question paper. Most of the questions are already known to the students as these are directly chosen from the Xth text book. Moreover huge option is available. Known nature of the questions allow the students to score undeserved marks by merely memorising the answers without any understanding of the subject. The available 'option' gives students the liberty to avoid D-level problems. Choice of questions from only Xth text book gives them the license to forget most of the concepts learnt in the earlier years. In spite of these unhealthy methods of studying, students do score very high percentage of marks. Unfortunately their faith in mindless mugging gets strengthened. The XIIth board examination being of the similar nature, their success(?) story continues. Finally and inevitably they meet their Waterloo at the first year engineering examination. If you do not learn from the failures of these seniors, I am afraid that you are heading to be the next Abhimanyu.(Arjuna's son in Mahabharata.)

Let us understand the difference between **Information and Knowledge.**
Let me share with you an anecdote. When I was is in XIth standard my Physics teacher wrote down three Newton's laws of motion on the blackboard. He made

us memorise these. Once he was satisfied that each one of us had correctly remembered these laws, he asked us "My Dear Students what have you learnt today?" All of us very proudly told him that we had learnt Newton's Laws. He laughed loudly. He then told us that we had merely memorised the statements of the Newton's Laws. He then spent next couple of months in explaining the meaning of each and every word in these statements and making us apply these in solving a number of practical problems. I am indebted to my teacher for bringing out the difference between information and the knowledge.

Let us have a close look at a typical student from standard VII. He computes simple interest using the formula $I = \frac{P \times R \times N}{100}$. During the course of the year he solves more than 50 similar problems. Let us carefully observe what he is actually doing. He is merely computing the product of P, R, N and dividing it by 100. He then checks this number with the number given at the back of the book. If it does not tally, he computes once again. Once his computations agree with the given answer, he solves one more similar problem, repeating this procedure all over again. All these efforts only ensure that the student has correct information about how to compute simple interest. To gain the knowledge he must understand why this formula really computes the simple interest. The proof, the derivation of the formula is really what mathematics is all about. If a student is not in a position to prove formula for the simple interest, do you think that he will be able to prove the formula for calculating the amount in the case of compound interest?

Check for yourself in the following situations whether you have only information or have the knowledge also.
(1) Heron's formula for calculating the area of the triangle.
(2) Rule for divisibility of a natural number by 9.
(3) Formula for the area of a trapezium.
(4) The method of constructing angle bisector of a given angle.
(5) The rules of Logarithm.
(6) The ratio of the circumference of a circle to its diameter is same for all circles.

Lokamanya Tilak had given us the slogan: "Freedom is my birth right and I will achieve it." On the same lines I want you to take the oath: **Knowledge is my Birth Right and I will acquire it. I will not accept any formula without proof. I will learn the justification of each and every geometric construction.** To implement this oath successfully you need to learn more about your most important instrument, the brain.

Understanding the way brain works.
Let me give you an analogy. I have more than 2000 books in my personal library.

If I buy a new book and arbitrarily place it somewhere then it will be very difficult for me to retrieve the book. I should certainly not keep a maths book along with the history books. Among the maths books, I must separately store the books on algebra, geometry, arithmetic, calculus etc. Among the geometry books, I should further classify according Euclidean Geometry, Co-ordinate Geometry, Differential Geometry, and Algebraic Geometry and so on. In our brain we have an astonishing amount of information stored. So any new concept we learn, we must be extremely careful in keeping it in the correct place in the brain. Now how do we really do it? If you study a new idea in isolation then in a very short time you will forget it. **You must make a conscious effort to associate the new idea with a number of similar ideas already existing in the brain.**

Whenever you study a new theorem follow as many of the following suggestions as possible.
(1) First try to think about, how the discoverer of this result might have found out this result. What experiments, he must have done which could have led him to this result.
(2) Try to prove this result independently. Who knows, your solution may be a much simpler one!
(3) If you do not get the proof, just read a couple of lines of the proof and with this starting point try once again.
(4) Once the complete proof is understood list all the theorems which you have used in proving this theorem. Prove each of these.
(5) Find out number of corollaries of this theorem.
(6) Find the various ways in which this result can be generalised.
(7) Teach this theorem to your friends.
All these steps will ensure that you have placed this new theorem in a correct place inside your brain.

Now you are bound to ask me **"How will we find so much time to do all these things?"** Let us return to the example of simple interest. Instead of wasting time in mechanically solving around 100 problems, learn to derive the formula. You will be surprised to discover that its proof is so simple. Having mastered the proof you won't find any need to solve more than 5 to 10 typical problems. Now you still have a lot of time which you could utilise to understand the concept of compound interest and derive the formula for amount in this case. You could create a number of formulae in the case of compound interest according as the interest is paid after each month or each day or even after each hour. You could relate this concept to population growth problem and estimate the future population of a given society. Hey! Probably you have already started enjoying mathematics. Having mastered the topic you could now very quickly scan through all the problems in the text book and identify that actually there are only 4 to 6 type of different problems in this chapter. This ability of **sifting the essence from the gross** is the first indicator of your future success.

What is the next step? In most of the schools, under the pretext of the revision the teachers will ask you to solve these 100 problems all over again. Solving these problems all over again is essentially going to be an exercise to the hand and not to the brain. **Rebel!** Refuse, politely but firmly, to do this mindless homework. Though India is a free country, you must earn your own freedom. Use this time to **Discover the creativity within yourself.** Try to construct some new problems on the topic you have completed from the school text book. Hold a competition among your friends and give a prize to the best discovery of the week. **Go beyond school text books.** Join the Library of Bhaskaracharya Pratishsthana or British Council Library. Subscribe to journals like Samasya, Resonance. Participate in the **Mathematics Olympiad Movement.**

Reference Books: Here is a list of some of really wonderful books you should include in your personal library.
(1) **Mathematical Circles: A Russian Experience. Author: Fomin.** Every student interested in Mathematics should have a personal copy of this wonderful book. (Rs. 195)
(2) Challenge and thrill of pre college mathematics.
Author: Krisnamurty. (Rs. 260)
(3) An Excursion in Mathematics.
Published by Bhaskaracharya Pratishsthana. (Rs. 65)
(4) Adventures in Problem Solving. Author: Shailesh Shirali. (Rs. 200) He has written many books. All are very good.
(5) Elementary Number Theory. Author: David Burton. (Rs. 175)
(6) Aisee Prameye Rasike. (in Marathi) Author: Ravindra Bapat. (Rs. 175)
Here is a list of some world famous authors. Try to read as many books written by them. (a) Yaglom (b) Polya (c) Coxeter (d) Ross Honsberger (e) Sharygin (f) Martin Gardner. All these books are available at

I sincerely hope that you will put to use various ideas I have discussed here. I am sure that you will start enjoying studying as never before. Wish you all the best in your exploration of this beautiful universe.

M. Prakash

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Hello Friends,

Our method of teaching mathematics to students at M. Prakash Academy is very different. Our experience tells us that students enjoy learning and excel in mathematics with this method.

We would like this method to reach many more students. So we are publishing our lecture notes of the lectures conducted for 8^{th} standard students on our website. New lectures will be added every week.

Do visit *http://www.mprakashacademy.co.in* and use these lecture notes. Please feel free to circulate these notes among your friends if you want to.

Current way of learning Mathematics	**Our vision of learning Mathematics**
Emphasis on short-term memory.	Emphasis on understanding the subject and appreciating it.
Cramming information.	Acquiring knowledge.
Makes students remember formulae.	Makes students recognize logic behind the formulae.
Proofs are demons; somehow remember them.	Make friends with Proofs. Play with them, appreciate their elegance.
Joy of learning is lost in copying readymade solutions.	Teachers provide impetus and students enjoy discovering solutions on their own.
Textbooks are sacrosanct. Don't question them.	Everything can be questioned. Textbooks are no exception.
Content of textbook is the ultimate limit. Guides and workbooks are the only extra material.	Content of textbook is the point of takeoff. Exposure to internationally renowned books. Real life experiences are also 'learning material'.

Please find on the following pages some elegant proofs learnt by our 8^{th} standard students.

We give an interesting proof of Pythagoras Theorem.
Following tools are used to prove the theorem.

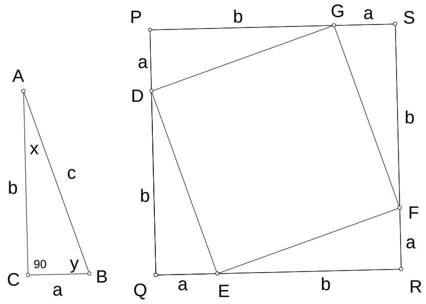
Tools:

(1) Side-Angle-Side (SAS) test of congruence of triangles.
(2) Sum of the angles forming a linear pair equals $180°$. Also the sum of the angles forming a linear triple equals $180°$.
(3) Sum of the angles of triangle equals $180°$.
(4) $(a+b)^2 = a^2 + 2ab + b^2$.
(5) Area of the triangle is half base into height.
(6) Congruent triangles have equal areas.
(7) A quadrilateral is called a **rhombus** if all it's sides have same length.
(8) A quadrilateral is called a **square** if all it's sides have same length and all it's angles have same measure equal to $90°$.
(9) Area of the square equals square of the length of its side.

Theme: Calculate the same area in two different ways.

Proof of the Pythagoras Theorem.

Statement of the theorem:
In $\triangle ABC$, if $\angle C = 90°$, then $a^2 + b^2 = c^2$ where $AB = c$, $BC = a$, $CA = b$.
Notation: Let $\angle CAB = x$, $\angle ABC = y$.
Abbreviation: (1) C.S.C.T.- Corresponding Sides of Congruent Triangles
(2) C.A.C.T.- Corresponding Angles of Congruent Triangles



Construction:
(1) Construct a square $PQRS$ whose each side equals $a + b$.
(2) Choose points D, E, F, G as shown in the figure such that $PD = QE = RF = SG = a$.
As each side of $\square PQRS$ has length $a+b$, we note that
$DQ = ER = FS = GP = b$.
(3) Join DE, EF, FG, GD.

Proof:
We note that each of the triangles $\triangle GPD$, $\triangle DQE$, $\triangle ERF$, $\triangle FSG$ is congruent to $\triangle ACB$ by SAS test....(1)
$\therefore$ C.S.C.T. gives $GD = DE = EF = FG = AB = c$.......(2)
At this stage, we have proved $\square DEFG$ is a **rhombus**.
C.A.C.T. gives
$\angle DGP = \angle EDQ = \angle FER = \angle GFS = \angle BAC = x$......(3)
$\angle GDP = \angle DEQ = \angle EFR = \angle FGS = \angle ABC = y$......(4)
In $\triangle ABC$, $x + y + 90° = 180°$, hence $x + y = 90°$............(5)
Straight angle $PDQ = \angle GDP + \angle GDE + \angle EDQ$
$\therefore 180° = y + \angle GDE + x$.....by (3) and (4)
$\therefore 180° = 90° + \angle GDE$..........by (5)
$\therefore \angle GDE = 90°$... (6)
Similarly, it can be shown that in $\square DEFG$,
$\angle GDE = \angle DEF = \angle EFG = \angle FGD = 90°$.....(7)
By (2) and (7) $\square DEFG$ is a **square**.
Let [PDG] denote area of $\triangle PDG$ etc. We will compute area of square $PQRS$ in two different ways. We have,
$[PQRS] = [PDG] + [QED] + [RFE] + [SGF] + [DEFG]$
$\therefore [PQRS] \quad = 4[ABC] + [DEFG]$ .....by (1)
$\therefore (a + b)^2 \quad = 4(\frac{1}{2}ab) + c^2$
$\therefore a^2 + 2ab + b^2 = 2ab + c^2$
$\therefore a^2 + b^2 \quad = c^2$.
This completes the proof of the Pythagoras Theorem.

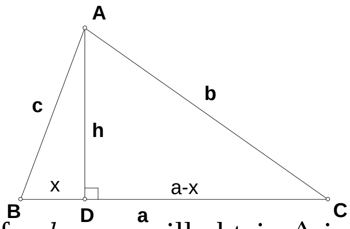
Derivation of the Heron's Formula.

Aim: To find the area of a triangle in terms of its sides.

Notation: In ΔABC, AB=c, BC=a, CA=b. $s = \frac{a+b+c}{2}$ denotes semi-perimeter and Δ (read as delta) denotes area of ΔABC.

Construction: Draw $AD \perp BC$.

Let AD=h, BD=x. Note $DC = BC - BD = a - x$



Tools: (1) Pythagoras Theorem (PT).

(2) Factors of difference of two squares.

(3) Area of triangle is half base into height.

Derivation:

We know $\Delta = \frac{1}{2}ah$. If we can calculate h in terms of a, b, c we will obtain Δ in terms of a, b, c.

As $\angle ADB = 90°$ applying PT to ΔADB we get,

$c^2 = h^2 + x^2 \therefore h^2 = c^2 - x^2$...(1)

Also as $\angle ADC = 90°$ applying PT to ΔADC we get,

$b^2 = h^2 + (a - x)^2 \therefore h^2 = b^2 - (a - x)^2$...(2)

(1) and (2) gives us $c^2 - x^2 = b^2 - (a - x)^2$,

$\therefore c^2 - x^2 = b^2 - (a^2 - 2ax + x^2)$

$\therefore c^2 = b^2 - a^2 + 2ax$

$\therefore c^2 - b^2 + a^2 = 2ax, \therefore x = \frac{a^2+c^2-b^2}{2a}$.....(3)

Substituting (3) in (1) we get,

$h^2 = c^2 - \left[\frac{a^2 + c^2 - b^2}{2a}\right]^2$. Now multiplying throughout by $4a^2$,

$$
\begin{aligned}
4a^2h^2 &= 4a^2c^2 - (a^2 + c^2 - b^2)^2 \\
&= (2ac)^2 - (a^2 + c^2 - b^2)^2 \\
&= [(2ac) + (a^2 + c^2 - b^2)]\,[(2ac) - (a^2 + c^2 - b^2)] \\
&= [2ac + a^2 + c^2 - b^2]\,[2ac - a^2 - c^2 + b^2] \\
&= [(2ac + a^2 + c^2) - b^2]\,[b^2 - (a^2 - 2ac + c^2)] \\
&= [(a + c)^2 - b^2]\,[b^2 - (a - c)^2] \\
&= [(a + c) + b]\,[(a + c) - b]\,[b + (a - c)]\,[b - (a - c)] \\
&= (a + c + b)(a + c - b)(b + a - c)(b - a + c) \\
&= (2s)(a + b + c - 2b)(a + b + c - 2c)(a + b + c - 2a) \\
&= (2s)(2s - 2b)(2s - 2c)(2s - 2a) \\
&= 16(s)(s - b)(s - c)(s - a)
\end{aligned}
$$

$\therefore\ \frac{1}{4}a^2h^2 = (\frac{1}{2}ah)^2 = (s)(s - a)(s - b)(s - c)$

$\therefore\ \Delta^2 = (\frac{1}{2}ah)^2 = (s)(s - a)(s - b)(s - c)$

$\therefore\ \Delta = \sqrt{(s)(s - a)(s - b)(s - c)}$ ...

This is **Heron's Formula**.