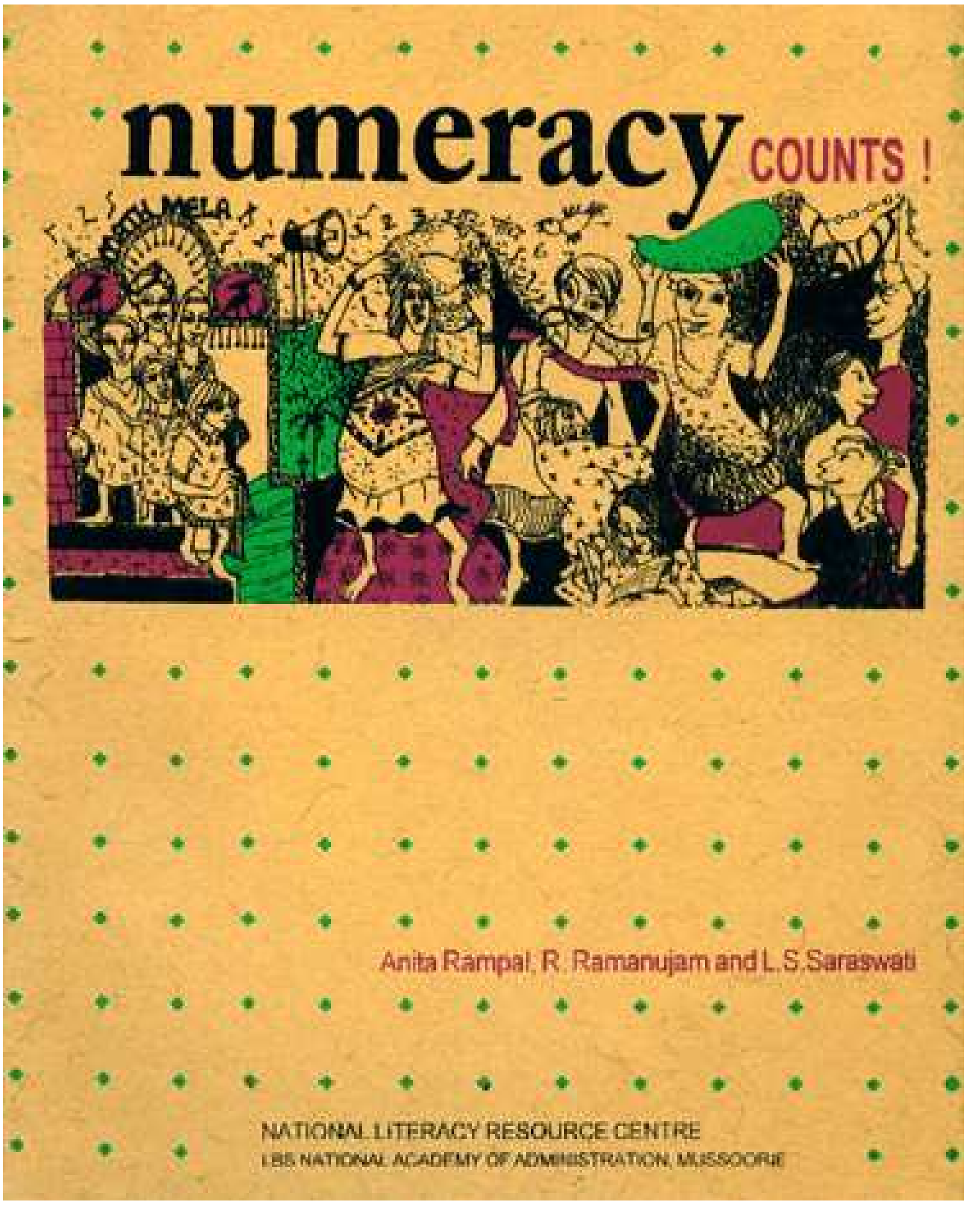

NUMERACY COUNTS!

Anita Rampal, R. Ramanujam and L.S. Saraswati

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National Literacy Resources Centre

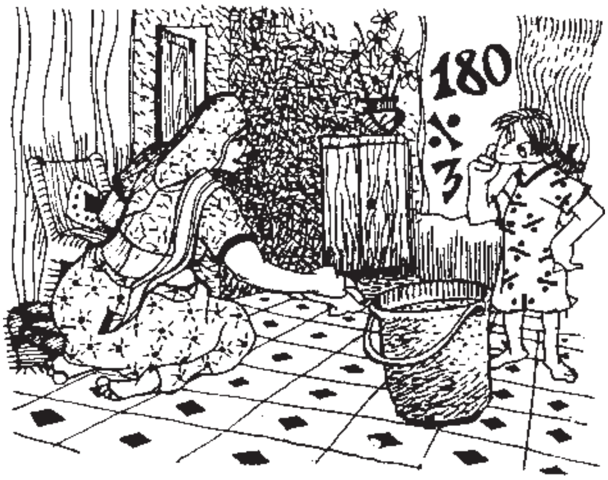
Another Way of Doing Sums!

The Red Queen remarked: I dare say you've not had many lessons in manners yet?"
"Manners an* not taught in lessons," said Alice, "Lessons teach you to do sums, and things of the sort."
"Can you do Addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one and one?"
"I don't know," said Alice. "I lost count."
"She can't do Addition," the Red Queen interrupted. "Can you do Subtraction? Take nine from eight."
"Nine from eight I can't, you know," Alice replied very readily: "but..."
"She can't do Subtraction," said the White Queen. "Can you do Division? Divide a loaf by a knife -what's the answer to that?"
"I suppose ..." Alice was beginning,
but the Red Queen answered for her, "Bread-and-butter, of course. Try another Subtraction sum. Take a bone from a *dog:* what remains?"
Alice considered. "The bone wouldn't remain, of course, if I took it - and the dog wouldn't remain: it would come to bite me ... and I'm sure I shouldn't remain!"
"I think that's the answer."
"Wrong, as usual," said the Red Queen: "the dog's temper would remain.
"But I don't see how ..."
"Why, look here!" the Red Queen cried. "The dog would lose its temper, wouldn't it?"
"perhaps it would." Alice replied cautiously.
"Then if the dog went away, its temper would remain!" the Queen exclaimed triumphantly.
"She can't do sums *a bit!"* the Queens said together, with great emphasis.

[From: "Through the Looking-Glass" by Lewis Carroll]

Chapter 1

Introduction

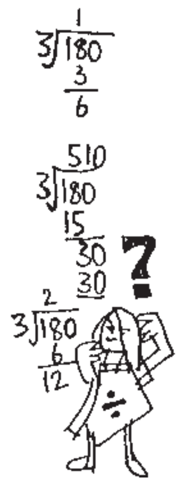


School-going Sunita and her non-literate mother Draupadi respond differently to the same oral problem.

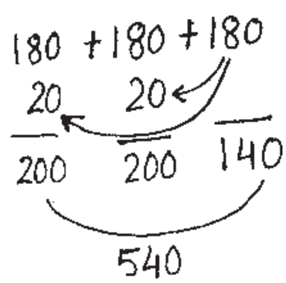
Verse is Worse

Multiplication is vexation,
Division is bad,
The Rule of Three,
it puzzles me
And Practice drives me mad!

(From David Wells, *The Penguin Book of Curious and Interesting Mathematics*)



Sunita, a young girl studying in class 5, is refusing to go to school because her teacher beats her for not doing her maths properly. She has been told she 'has no brains' and had better go to work with her mother. Her mother Draupadi earns her living as a domestic worker, sweeping floors, washing clothes and utensils, running from house to house throughout the day, and is very keen to educate her daughter so that she is not confronted with a similar fate. On talking to Sunita we find she is unable to do her sums at school and says she has problems making sense of' division'. "But you know how to do division, isn't it? See, if your mother wants to divide Rs. 180 equally between her three children how much would she give you?" we ask her reassuringly, expecting her to give a pat reply. Sunita seems confounded and stares blankly, but her mother, unschooled and barely literate, laughingly provides the answer instantly. On probing further it becomes clear that Sunita is totally confused by the algorithms she has been taught at school, and is unable to use her living experience to mentally operate on numbers the way her mother can effortlessly do. For instance, when the problem is stated in words Sunita first desperately tries to translate it into some mathematical operation between the numbers 180 and 3, and often cannot decide which one it should be, the 'x' or the '+', or some other? Even when she is told she has to divide 180 by 3, she can only picturise the school method of writing numbers in some meaningless strange pattern of long division', and soon gives up.



This is how Draupadi mentally multiplies 180 by 3

> It has well been said that the highest aim in education is analogous to the highest aim in mathematics, namely, to obtain not *results* put *powers*, not particular solutions, but the means by which endless solutions may be wrought.
> George Eliot

Draupadi, on the contrary, is confident of handling such mathematical operations in her daily life, uses many different mental algorithms depending on the problem posed, and fortunately did not suffer from the 'school syndrome' which casts most children as 'having no brains'. In a flash she has broken the problem into manageable parts, has first distributed 50 to each of the three children, and then of the remaining 30 she has again doled out 10 each, so that in the end each has been given Rs. 60. Had we asked her to divide 190 she would continue the same process iteratively, with an extra 10 to distribute among three. We next ask her how much she would need in all if she had to give Rs. 180 to each, and again promptly, almost without batting an eyelid she smilingly says "Rs. 180 for three of them? Rs. 540". Amazing, without knowing any tables and without being able to write these figures! How did she calculate? Of the last 180 she mentally gave 20 to each of the first two and then added 200 + 200 + 140. Another amazing observation is that she hardly ever goes wrong, and has sound mechanisms of self-checking her answer if ever she is in doubt. Indeed, she proudly confesses that she has often challenged her husband, a skilled literate mason, that she can perform more complicated mental computations faster than he can by his written methods!

This is not an isolated example of some 'slow learner' and her 'bright' mother. In fact, this is the case with the majority of our children who might enroll into school but are soon forced to drop out because they are unable to cope with the demands of school and find studies 'uninteresting' or 'difficult'. The way teaching is designed by their textbooks and teachers is alienating and strange for them, not allowing them to use their rich life experiences, and they are soon made to believe that they 'have no brains'. Children who dropped out early or were never enrolled in school grow up to constitute the large numbers of adult non-literates in our country (as also in other developing countries). It is now becoming clear that if our teaching strategies in elementary school had been more sensitive and relevant in these past fifty years we would not have been faced with such a stupendous task of making millions literate through Mass Literacy Campaigns.

Draupadi is also not exceptional; most adults are engaged in everyday mathematics and are often capable of sophisticated mental computations. In fact, oral numeracy is more natural among unschooled adults than literacy, which means that they are more familiar with numbers, operations, measurements, etc. than with letters. However, the strategies they use in oral arithmetic are often very distinct from the usual methods of written arithmetic, (see chapters 3 and 4 on 'oral and written arithmetic' and 'folk mathematics').

Just as we do not start by teaching adults how to speak before teaching them to read and write, we do not have to teach them to count simple numbers or to add before teaching them written numeracy. Adults are aware of the limitations of using only their memory for keeping numbers in their mind; especially when they tend to lose track of sub-totals while doing complex calculations. They would find it a great help to be able to write numbers, but they do not want to be taught only how to write 1 or 8 or 22. They quickly want to learn to write larger numbers, and our numeracy programme must reach this level of teaching useful skills at an early stage. We must also reinforce their mental arithmetic skills (instead of trying to replace or ignore them), and help them with different types of record keeping which are of practical use in their daily lives. Unfortunately, our primers do not seem to respect this fact.

A major problem is that those of us who design primers and teaching methods for adults are totally unaware of their strategies and insist on imposing on them routines which make no sense to them and often end up in boredom, frustration and failure, similar to the 'school math syndrome' suffered by millions of children.

An International Problem

The mismatch between teaching and learning strategies for mathematics is not just limited to our own efforts but has been felt in many other literacy campaigns and adult education programmes all over the world. As part of the International Literacy Year in 1990, there were many critical essays and reviews of crucial issues encountered during different literacy campaigns undertaken by various countries. In particular, a report on "Arithmetic in Daily Life and Literacy" by Claude Dalbera, brought out by UNESCO under its series 'Literacy Lessons', attempted to highlight that arithmetic or mathematics was still a weak area and needed much more concerted effort from ail concerned parties. It would be interesting to restate some of those concerns here since they are strikingly similar to the problems we continue to face in our own campaign. It also shows that our 'academic experts' who formulated policies for our primers and pedagogies are pretty slow learners in picking up even the essentials from these 'literacy lessons' based on numerous earlier experiences!

By 1990 (around the time we began our Literacy Campaigns) people all over the world had accepted the value of participation', based on the actual concerns and experiences of nonliterate learners. Therefore, as far as reading and writing were concerned there seemed to be a clear understanding that teaching methods must proceed to codify speech and verbal thought, not start mechanically from alphabets. But what about the reading and writing of numbers? Arithmetic? The problems involved with money matters, measurement, etc. How do we deal with these?

The UNESCO report points out that (we reproduce almost verbatim from the document):

> Oral numeracy is more natural among unschooled adults than literacy, which means that they are more familiar with numbers, operations, measurements, than with letters.

> In many campaigns, it has been found that the literacy portion is progressive but the numeracy section remains old fashioned and childish, causing adults to drop out of class.

Simply to ask these questions can be baffling. There are relatively few literacy programmes which have succeeded in defining methods of learning and thinking in these fields which are adapted to adults. For example, only too often we find literacy programmes which include a highly 'revolutionary' section aimed at providing access to written speech, increasing social awareness, encouraging development, etc. but a highly 'reactionary' section providing access to written arithmetic. Adults find themselves faced with an old-fashioned method directly traceable to traditional primary education, childish and superficially adapted to the environment (marbles may have been replaced by eggs to create a 'rural' impression), leading to poor results or none at all.

This has sometimes caused adults to abandon their literacy classes. Faced with such difficulties, therefore, some teachers responsible for literacy progrmmes simply delete the 'arithmetic' section from their programmes.

Fortunately, as soon as the principle of participation' is adopted, non-literates do not hesitate to express their needs loudly and clearly. Many state that learning to count and calculate, and thinking about the important socio-economic problems connected with them, are a high priority for the future of their families and their community. Hence the importance of keeping the discussion on arithmetic open by exchanging experiences and ideas and by spreading new methods. It must be admitted that although literacy in general has, generated a great amount of specialized literature, this is not the case with arithmetic in particular.

An arithmetic programme must be able to transform the principal needs felt by adults into specific educational methods and content, by going directly to the most essential and useful matters. And quickly, for adults have only a limited time for learning. With them it is not possible, as it is at the school level, to put off the acquisition of more satisfying skills to a later time. Especially since the mastery of arithmelic offers new practical tools which can supplement the mental techniques already used by them. Both to save time and be realistic, it is essential to start from what adults already know, and enable them to develop their ability to cope with their problems themselves.

We must not spend too much time on basic concepts as done with children in school, but must go directly and quickly to the more essential and satisfying skills.

Only 40% of seventeen year old Americans could answer: What is 87% of 10? How many of us can tell if the square root of 17 lies between 9 and 10 or, 17 and 18 or, 4 and 5?

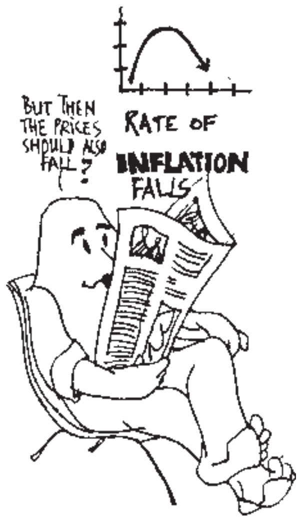
As we know non literates are almost always the poorest people, the most exploited and the most oppressed. They cannot be expected to agree to learn to read and write out of blind loyalty to moral or civic considerations or out of free devotion to some noble cause for their benefit. In order not to waste their time literacy must offer them a real opportunity to help change their situation. So-called educational activities which do not take into account the aptitudes and knowledge from their experiences in personal and community life cannot ultimately help them advance towards economic, political, social and cultural emancipation.

(from the report *"Arithmetic in Daily Life and Literacy"* by Claude Dalbera, brought out by UNESCO: International Bureau of Education, 1990)

Innumeracy in Industrialised Countries

The last decade has witnessed a loud cry from both the U.K. and U.S.A about large numbers of innumerate citizens who suffer from deep maths anxiety and are incapable of solving simple arithmetical problems. Despite having achieved universal schooling these industrialized countries are expressing great concern about the level of mathematical performance of their adults, and are now questioning the efficacy of their school mathematics teaching. The study in U.S.A. which created quite a ripple was the 'The Mathematics Report Card' (1988), which had sampled 80 percent of the seventeen-year-olds still at school. It revealed that only 40 percent of the nation's students could solve moderately sophisticated problems such as finding 87 percent of 10 . Only 6 percent of these students, representing only one in twenty of young American adults, could perform simple multi step problems such as calculating total repayment (principal plus interest) on a loan, or locating the square root of 17 between two consecutive integers. These results had confirmed what the Cockroft Commission had reported in England some years earlier.

The British commission had interviewed hundreds of adults (in addition to conducting large scale written tests), to determine how they used mathematics on the job or in their daily lives. Interviewers had discovered a common perception of maths being such a "daunting subject" that more than half of those approached refused to participate in the study. The 'most striking feature' of the study was the extent of anxiety, helplessness, fear and guilt felt by common people in the face of simple problems. It documented widespread inability to handle percentages, even those used everyday in the context of tips or sales tax. Many adults thought that a fall in rate of inflation reported in newspapers should cause a fall in prices. The Cockroft study also discovered a surprising pattern. Most workers who needed to use specific job-related mathematics did so by methods and tricks passed on by fellow workers that had little connection with methods taught in school. Tradesmen



dealt in fractions with limited sets of denominators, such as halves, quarters, and eighths etc., rather than the 'common denominator' strategies taught at school. In one example, a worker who needed to frequently multiply numbers by 7 did so by multiplying by 3, adding the result to itself, and then adding the original number.

The paradox of workers learning or inventing new strategies instead of using those taught at school is the result of insecurity brought on by their school experience. 'Many otherwise well educated adults are virtually innumerate. Unless the mathematics studied at school is understood with confidence - and all data show that only a minority achieves this type of understanding - it will not be used in any situation where the results matter'
(Article by Lynn Arthur Steen, *in Daedalus,* vol. 119, p. 211-231).

During the last few years much effort has been made to understand strategies of learning maths which create confidence and an 'open' attitude to solving problems. A joint US-Japanese Commission on Mathematics Education has stressed the need to encourage learners using multiple ways of solving a problem. They found that Japanese students used such open ended strategies and performed much better than their American counterparts. In the case of adults, it has repeatedly been emphasized that the Math curriculum must be redefined to encompass a rich blend of art, entertainment and wholesome common sense.

Certain recommendations are given below;

• *Not to teach just arithmetic.*

Numeracy requires a rich blend of statistics, geometry, arithmetic and other pattern recognition techniques, catalyzed by careful reasoning rooted in common sense.

• *Not to rely on artificial lessons alone.*

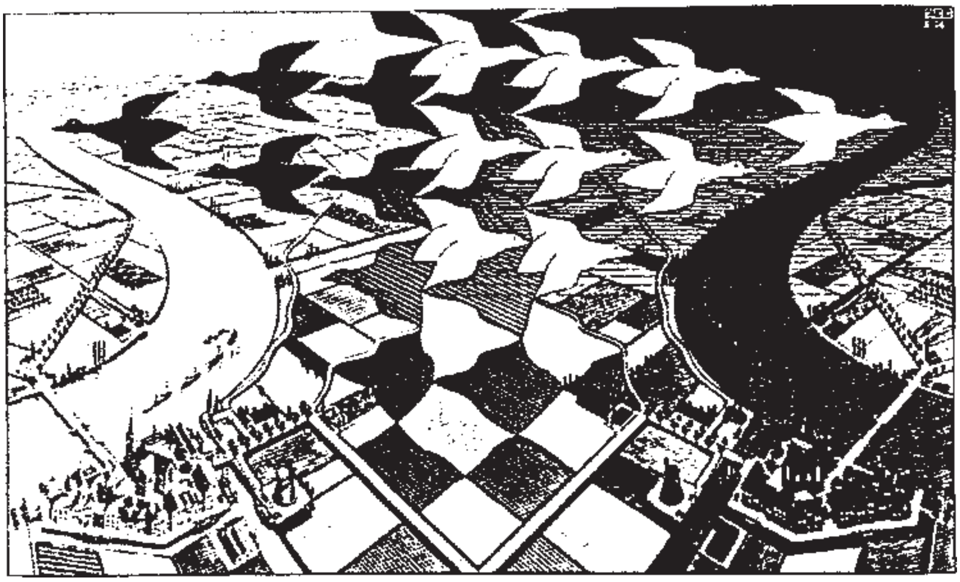
Learners learn best in active contexts featuring discussion, writing, investigation, and cooperation. Isolated lessons on artificial worksheets reinforce the image of school mathematics as totally alienated from everyday life.

In the case of adults, it has repeatedly been emphasized that the Math curriculum must be redefined to encompass a rich blend of art, entertainment and wholesome common sense.

• *Not to use just short-answer tests.*

Assessment instruments strongly influence the shape of teaching and learning. In numeracy as in literacy, formulation and expression are more important than getting the correct answers. Tests should be designed to reveal how learners think, not just what they know.

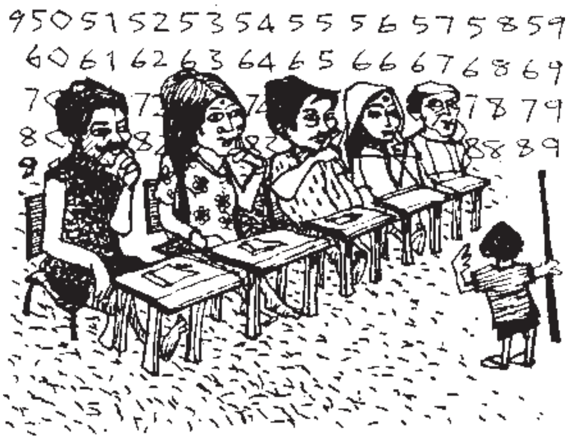
• *Do not depend only on mathematics.* Although numeracy may be taught in mathematics lessons, to be learned effectively it must be integrated widely in other contexts, in and out of class, in entertainment, through games and cultural activities.



In this drawing the squares at the bottom gradually transform Jo a set of white and black birds alternately flying in opposite directions. This woodcut print is by M.C. Escher, whose work has continued to fascinate mathematicians, the world over.

Chapter2

A Review of Numeracy in our Literacy Campaigns



Adult learners treated as children in primary schools (Quiet! Yes…fingers on your lips!)

It is widely acknowledged that achievement levels are fairly low with regard to numeracy in the Total Literacy Campaigns (TLCs) of most districts. There seems to be little systematic effort to remedy this situation during the post-literacy phase either. We briefly discuss possible reasons for this, before attempting to offer possible suggestions and solutions.

The Given Structure of Primers

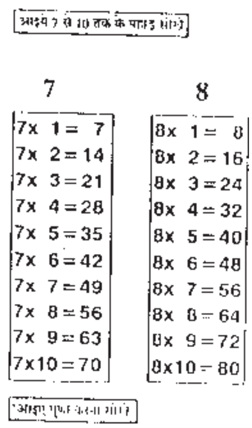
According to the norms defined by the National Literacy Mission, at the end of the TLC phase, each learner is expected to have acquired the following numeracy skills:

• Fluency with numbers up to 100.
• A thorough familiarity with the four arithmetical operations and an ability to manipulate simple fractions.
• Understanding measures of time, length, volume and weight in standard units.
• An ability to interpret quantitative data (facts and figures) encountered in everyday life.

The primers for these TLCs, designed according to the IPCL (Improved Pace and Content of Learning) guidelines, typically have the following structure:

• In the first primer, the numbers 0 through 50 are introduced, normally in steps of 1-10,11-20,21-30, etc.
• The second primer has numbers 1 through 100, and exercises in addition and subtraction. Clock time is also introduced.
• The third primer includes exercises in multiplication and division, measurement and basic concepts in decimals, fractions, money transactions, etc.

While this is largely true of primers in most districts, some have tried to include additional material involving percentages, interest calculations, etc.



7	8
7x 1 = 7	8x 1 = 8
7x 2 = 14	8x 2 = 16
7x 3 = 21	8x 3 = 24
7x 4 = 28	8x 4 = 32
7x 5 = 35	8x 5 = 40
7x 6 = 42	8x 6 = 48
7x 7 = 49	8x 7 = 56
7x 8 = 56	8x 8 = 64
7x 9 = 63	8x 9 = 72
7x10 = 70	8x10 = 80

A typical page from Primer 3 showing how numbers are routinely presented in a dry and distant manner

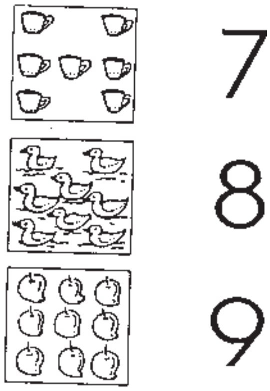
Problems due to the Primer Structure

The primer structure outlined above is definitely ill-advised. It is clear that adults are treated as children, and are taught slowly and often painfully linearly in a manner normally adopted in school. The IPCL Committee has deliberated over the dismal achievements in numeracy, and has ironically ‘diluted’ the curriculum each time, in a somewhat misguided attempt to make it’ simpler” for adults. Unfortunately many of the traditional notions of formal mathematics education hold sway, and there is little effort to understand the exigencies of a short time-bound campaign approach. For instance, one expert had struck off all attempts by an urban district TLC team to include 'functional' topics of interest to its learners, such as dates on a calendar, on birth certificates or on medicine packets, etc. and had insisted that even in Primer 2 numbers be taught from 51-60, 61-70, etc. in each successive chapter!. The basic fact that learners, especially from urban backgrounds, were in a hurry to come to the more 'useful' skills and knowledge, and would quit if made to plod through simple numbers in a boring childish fashion was impossible to communicate to the committee members.

The process of learning in an adult is not linear and the adult is already familiar with a lot of mathematical knowledge from everyday life experience. The decision to have numbers up to 50 in one primer and the next 50 in another makes little sense. A number like 87 may seem incomprehensibly large to a child but not to an adult. In terms of place value, why should a learner who can distinguish 23 from 32 have any trouble at all with 67 and 76? Thus, in what sense are the numbers 51 through 100 'more difficult' than 21 through 50, and fit to be relegated to Primer 2?

Why should one be familiar with addition/subtraction up to 100 before learning multiplication? Such questions need to be thoroughly reviewed and discussed in the light of our vast experience of running large literacy campaigns.

A typical page from Primer 1. Pictorial illustrations which give 8 ducks and ask learners to count how many ducks there are constitute tin insult to the learner's intelligence. It is then no wonder that learners don't lake numeracy lessons seriously at all.



Primer 1 is indeed a study in how insensitive we can be to what learners already know. Pictorial illustrations which give 8 ducks and ask learners to count how many ducks there are constitute an insult to the learner's intelligence. It is then no wonder that learners don't take numeracy lessons seriously at all.

Arithmetical operations such as addition, subtraction, etc. make sense to learners as contextual problems, which can be stated in words and similarly solved in words. Just as Draupadi managed to solve the problem of division effortlessly in a real life context, adults routinely encounter such transactions and have often evolved their own strategies. We should therefore start by discussing 'word problems' and gradually go on to the written form as abstract representations of those.

Stating arithmeticalproblems in words keeps the context alive and provides a real world link for adults, so that they can make sense of the 'jugglery' with numbers. Unfortunately our own primers hardly contain such word problems.

Stating arithmetical problems in words constantly *keeps* the context alive, and provides a real world link to adults, so that they continue to make sense of the jugglery with numbers (at which they are far better than us when it comes to mental operations in a real context!) However, we have found that our primers hardly deal with 'word problems'.

In fact, our experts naively believe that word problems are 'too difficult' and hence must be avoided. This is a deep rooted myth that also stems from our school system. However, even in school if word problems were not as 'formal' as they normally are but were designed to encourage more mental mathematics embedded in meaningful contexts, with various open and informal' strategies that children use, we would find the response to them very different, (see Chapter 3 on 'Oral and Written Arithmetic' for more discussion on 'word problems')

We are aware of the sense of alienation suffered by children; thanks to the style employed in school textbooks. The situation seems no different when it comes to TLC primers with regard to numeracy. The primers completely ignore the fact that adult learners have a sound experience in dealing with quantities of everyday life.

Calculations involving money, in particular, profit and loss, interest on loan computations, or the probability of winning and losing in a lottery, are also perceived as important by learners themselves and need to be part of the numeracy curriculum, but these are typically not addressed in TLCs.

Practical Problems during the Campaign

As a direct result of this faulty structure various practical problems arise in the campaign itself, some of which are as follows:

• Initial enthusiasm, in terms of rounding up volunteers, enrolling learners, enlisting resource persons is very high. Thus, major training inputs, particularly on the academic front, relate mostly only to Primer 1, where the numeracy component is trivial.
• All along the training line from key resource persons (KRPs) to volunteer teachers (VTs), nobody takes numeracy seriously, since the primer seems to trivialize it.
• Teaching starts, and the transaction of Primer 1 takes 4 to 5 months. Numeracy is no problem at all' is the cry 'from the field'.
• When training for Primer 2 is conducted, there is even less seriousness about numeracy, thanks to the perception that this is hardly worth bothering about.
• By the time cracks appear, typically with adding numbers like 28+34, classes are well into one half of Primer 2, and we are now about 9 months into the campaign (since teaching started).
• By the time the perception is shared and a conclusion is reached, that 'numeracy is weak', nearly half the learners and VTs have dropped out, there are few resource persons (RPs) left, and field co-ordinators who double as RPs have no clue what to do.
• The few who participate in Primer 3 training as well as teaching simply ignore the numeracy part owing to the conviction that it is 'too difficult'.
• Finally we hear, 'numeracy is really weak', and in a couple of campaigns (very few, in fact), some extra effort is put in during Post Literacy Campaign (PLC). However, the same mistakes are repeated, yielding the same result. In all this, the rushed training in TLC is certainly to blame, but the main culprit is indeed the structure of the primer.

We need a much more meaningful way of dividing content into different primers, taking into account the kind of TLC dynamics described above.

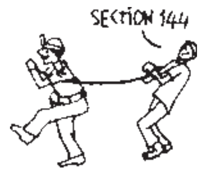
A Pedagogical Problem : Numbers too have Meaning?

In the pedagogy of literacy, much emphasis is laid on a word-based approach to introduction of letters rather than the direct introduction of alphabet. The writings of Paulo Freire have influenced the thinking of many pedagogues on this issue. The crucial point, seems to be that the bottom-up approach lacks meaning, and words loaded with life form better vehicles to understand letter-structure.

> "Literacy education for adults becomes an introduction to the *effort to systematise the knowledge* that rural and urban workers gain in the course of their daily activity. The *knowledge* of earlier knowledge then opens to them the possibility of new knowledge."
> Paulo Freire, *Pedagogy in Process,* 1978

For some reason, this point is totally forsaken in the context of numeracy. Presumably, individual numbers are still meaningful by themselves (though the numerals are not), so no more is said. But such an argument only misses the point that unless we approach numbers in terms of what they mean to the learner, what numerical operations signify to learners, we are wholly skipping Freire's methodology applied in the case of teaching letters.

What can numbers mean to learners? Once a group of learners were asked what hundred objects they could think of when they heard the word '100'. Many thought of flowers, some thought of mangoes and so on. When they heard '1000' it was mostly a thousand rupees. However, when it came to '144', there was a stunned silence. One woman, after some thought, said "144 benches in a cinema hall". This emphasizes the point that numbers are most meaningful when they evoke a mental picture in the learner. And most adults (even literates like us!) thoroughly enjoy doing such exercises. If only we followed this basic principle in introducing written numbers, rather than make it a meaningless ritual that goes on and on, and on.....almost through two primers!



The Role of the Volunteer Teacher (VT)
The 'Contract' Problem

A major feature that distinguishes adult education from children's education is that the former is effective only when there is an explicit (though unarticulated) contract between the teacher and the learner, as follows :

• the learner is aware that she lacks a particular skill, say, the ability to read and write.
• the teacher begins with the premise that the learner lacks this 'identifiable' ability.
• the instruction programme is an undertaking to impart the skill.

In some sense, drop-out phenomena in TLCs can be explained by the unfulfilment of such contracts. The expectation on the teacher's part of the task she must perform also shows up in her inability to cope with learners who already have some basic knowledge. This is also related to the often-heard complaint about learners being at different levels in the same class.

When it comes to adult numeracy, there is a special problem. As we have mentioned earlier, most learners are familiar with numeracy in their daily life transactions even though they do not know how to read or write. In actual fact, they may even be more efficient at mental calculations or traditional measurements then the young VT, just as Draupadi was as compared to her 'schooled' daughter (mentioned in the first chapter). Thus a characteristic feature of numeracy in the TLCs is the lack of a clearly defined contract. The definition of numeracy according to the NLM may be precise enough, but:

> In case of literacy the teacher can begin with the premise that the learner lacks the 'identifiable' ability to read and write. But when it comes to numeracy, adults already have some skills through their daily oral calculations.

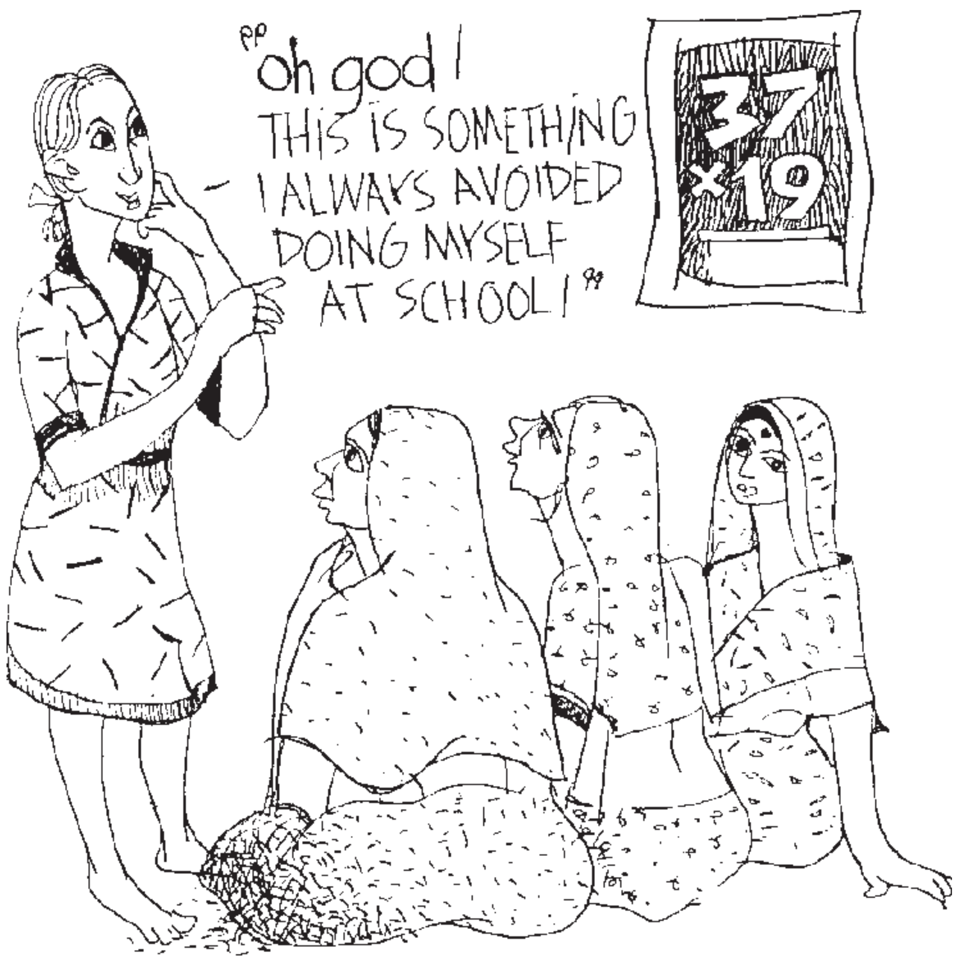
• The learner has no clear idea of how much arithmetic she already knows and what she does not know.
• The teacher has no clue at all about what the learner needs to know, but does not know.

In the absence of such a contractual obligation, mere textbook transaction results, rather more like a ritual than any meaningful instruction.
In TLCs we have the peculiarity that often the volunteer teacher is himself semi-literate and barely manages

to read and write comfortably on his own. Generally the VT is one who is in school or has been to school, perhaps up to class 8. Despite such inadequacies, neither he nor the learners in his class consider it a serious handicap for most of the literacy curriculum, especially if they have been trained.

When it comes to numeracy, the situation is seriously worse. At most times, the VT is/was deeply scared of math at school, having personally suffered from the typical ‘school math syndrome’. Many of the ‘problems’ like two-digit addition/subtraction, or worse, multiplication and division, seem hard stuff to the VT too. When it comes to ‘word problems’, he is at his anxious worst.

We thus have a situation where the VT has no clear understanding of what the learner knows already, what she needs to know but does not, and how capable he himself is of imparting the required knowledge. In such a pass, what can be expected in class? The ‘difficult’ portions are conveniently skipped! Ironically, those who train the VT are typically school teachers who make the same mistake as they do at school. They simply ignore his math anxiety, are unable to draw upon his and his learner’s own life experiences, so that ultimately the VT gets little of the necessary inputs during training.
All this calls for drastic revision in the way the numeracy component is treated in the TL/PL campaigns, both at the level of primers and also during trainings of all persons involved. It is hoped that the ensuing discussion will pave the way for such a revision.



Chapter3

Oral and Written Arithmetic

In the opening section we had marveled at how Draupadi performed mental calculations to solve everyday problems. There are clear differences in the procedures of oral and written arithmetic. Here we shall look carefully at other examples and try to understand the oral procedures followed by adult non-literates. We shall also refer to some studies conducted in other parts of the world to compare the strategies used by adults in everyday practices.

Example 1

Janaki sells vegetables at the weekly market. She needs to compute: 200 – 65

'I first give five so it is seventy, then eighty, ninety, hundred. I give back one hundred thirty five'.

This is a common practice adopted even by literate shopkeepers when they return change after any customer has paid a larger amount. Instead of subtracting they normally add up to the required amount. From 65 they first add up to 70 and then continue in multiples of 10. The 200 is decomposed into 100 and 100, and the problem is solved in parts.

Example 2

Bhairu needs to compute the following: 343 - 48. How does he do it?

'Take forty three, leaves three hundred; take five more; leaves two hundred ninety five.'

Bhairu broke up 343 into 300 + 43; 48 was also broken into 43 + 5; canceling both 43's, what remained was to take away 5 from 300. Now just think how you would solve this orally?

Example 3

Chandru, a school drop-out, is asked to solve: 200 – 45

When asked to do it in writing he writes the numbers one below the other. Then starts from the units to tens and hundreds, saying aloud *"five out of zero, nothing; four out of zero, nothing; two left, gives two hundred"*

A researcher asks how can it be - that if he had Rs. 200 to begin with and bought something for Rs. 45, he couldn't go back with the same Rs. 200?

The boy writes it down again and proceeds from units to tens to hundreds in the same way. This time he says aloud: *"zero out of five, five; zero out of four, four; leaves two on the left, gives two hundred forty five.... but this is also wrong."*

Next he is asked to compute this orally and he confidently replies: *"If it were fifty I'd get back one hundred fifty, but now I'll get one hundred fifty five".*

Keeping in Mind the Relative Value of Numbers

In written procedures we always start from the smaller quantities, that is the units, then the tens, then hundreds, etc. If there are any mistakes in the first step they amplify as we go on. In oral arithmetic we begin with larger quantities and only then proceed to fine tune the result by going to die smaller ones. As in example 3, orally the boy begins by subtracting fifty from two hundred, and only later makes the correction by considering the additional five. This is one of the reasons why errors are normally smaller in oral arithmetic.

$$\begin{array}{r} 200 \\ -\,45 \\ \hline 200 \end{array}$$

$$\begin{array}{r} 200 \\ -\,45 \\ \hline 245 \end{array}$$

In written arithmetic we do not need to think about the relative value of the numbers involved, since we look at only the relative positions, such as, units, tens, hundreds, etc. For the number 333 we orally say 'three hundred and thirty three' but write it symbolically as 3 3 3, so that only the placement of each 3 matters. Working with the number orally we will always look at each of the 3s as different, depending on its relative value of three hundred' or 'thirty' or 'three'. Thus orally we always preserve the relative value of each number and manage to keep track of it throughout our calculations. Orally we do not have to look at the units separately, or the tens etc. This is why orally we are never faced with the situation as in example 3, where the boy was baffled by the statement 'five out of zero[1]. Remembering the rule for 'carry over' is also an additional burden and does not come very naturally in mental arithmetic.

Example 4

A neo-literate woman is asked to compute: 100/4

When asked to solve the problem on paper she found it impossible. She first tried to divide 1 by 4 but couldn't, then tried 0 by 4, and finally gave up.

Asked to do it orally, she said it was easy; because she first divided hundred by two and got fifty, and then again divided by two to get twenty five.

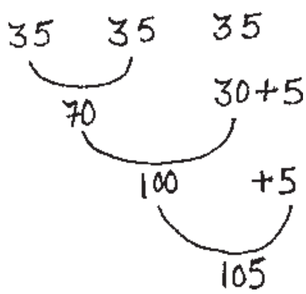
In oral arithmetic we first compute the larger quantities and only then proceed to fine tune with the smaller ones. This is one of the reasons why errors are normally smaller in oral arithmetic.

Example 5

A young boy in a grocery store is asked to compute: 35x3 (the price of 3 kilos of washing powder at Rs. 35 per Kg.)

On paper he writes 35 x 3 and says *'three times five is fifteen, carry one; one and three are four, three times four is twelve'.* The result written is 125.

Orally he does it as *'thirty five and thirty five is seventy, plus thirty is hundred, ...it is one hundred and five'.*



Repeated Grouping as a Useful Strategy

The algorithm or set of rules for 'long division', used in written arithmetic, is very difficult for most people (including school children) to learn and remember. It is not a natural method of computation used in everyday transactions, and requires a sequence of steps which do not necessarily have any 'meaning' associated with them. Division is done naturally by most people either by 'repeated grouping', such as was done by Draupadi (180/3 is 50 + 50 + 50 and then 10 +10 + 10 to give 50 + 10 in each group.) Or as in example 4, people also adopt the method of' factoring' so that two successive divisions by 2 replace the original division by 4.

Repeated grouping is used in oral arithmetic both for multiplication and division. It basically means multiplying by successive additions or dividing by successive subtractions. The groups are convenient chunks that can be easily dealt with, and the person keeps track of all these successive groups mentally. Sometimes fingers or other concrete objects such as stones are also used to keep track of the groups. In fact, this is why this procedure becomes difficult for very large numbers, where many successive steps have to be mentally kept track off.

A Study with Street Vendors: 'Formal' and 'Informal' Situations

We have seen that the same person uses different procedures while solving a problem orally or in writing. A systematic study was done in 1982 in a small town of Brazil, with five children (ages between 9-15 years), who attended school and also sold vegetables in the street market. A comparison was done to see how they tackled problems in the ' informal' setting of the market, while selling goods to researchers posing as customers, and later in a more 'formal' setting in their home, where they were given school-type problems to solve with paper and pencil. The formal test consisted of two types of problems - one were the 'pure computation' type (e.g. 'how much is 35 times 10?'). The other was word problems', which mentioned real objects and quantities (e.g. 'if one coconut costs 35 cruzeiros how much would be the cost of 10?'). In either case, the children were given formal problems using the same numbers involved in the problems they managed to solve successfully in the 'informal' test. All together they solved 63 problems in the' informal' test, and 99 in the' formal' test.

The results are similar to what we should now expect. The problems presented in the street were much more easily solved than ones given in school-like fashion. In the informal test they gave 98% correct responses. In the formal test they got 74% correct answers in the word problems, hut only 37% correct for the 'pure computation' problems. The relative success in the street could not be attributed to 'concrete reasoning', since the presence of real objects had not in any way reduced the mathematical demands of the problems or made them any easier. We shall quote from the conclusions drawn by the researchers:

> It could not be assumed, as educators often do, that applied or 'word' problems would be more difficult than pure computations. They appeared in fact to be easier.

'One of the implications we drew was that it could not be assumed, as educators often do, that applied or 'word' problems would be more difficult than pure computations. They appeared in fact to be easier. They also raise doubts about the pedagogical practice of teaching children first how to solve mathematical operations simply with numbers, and only later apply them to verbal and real life problems."

(From the paper by Carraher, Carraher and Schliemann (1985), 'Mathematics in the streets and in school', *British Journal of Developmental Psychology,* volume 3, p.21 -29)

If we look carefully we find such examples are all around us, happening all the time. While teaching adults it is even more important to deal with 'word' problems in real life contexts. In the methods they normally use they try to preserve the meaning of the problem in the given context. When asked to solve problems in writing people often make mistakes because the mechanical procedures used in written arithmetic do not compel them to look for any 'meaning' in the result they obtain. We shall elaborate on this point a bit more.

Oral Practices Preserve Meaning

If one has to add money such as Rs. 4.25 and Rs 2.90 by writing it down, we can simply proceed to add the numbers without really bothering about the decimal point. As long as the numbers are properly placed one over the other, the written addition procedure works just as well with or without the decimal point— We simply get the answer Rs. 7.15. However, adding orally 'four rupees and twenty five paise' to 'two rupees and ninety paise', one is forced to convert from rupees to paise and keep track of the relationship that 'a hundred paise makes a rupee'. Oral arithmetic thus requires conversions or recoding from rupees to paise and has to take into account the changes of units. This helps in keeping oral procedures close to meaning and involves more than just knowing the mechanical algorithms.

In contrast, written practices in arithmetic require only the rules to be known. Using the decimal point in written procedures provides [he advantage that the user need not know what changes of units take place after the decimal. The rules are general and can be blindly applied to get an answer for all cases, he it money, meters, or any other metric measurement. For instance, in writing, 2.5m can be straight off added to 1.75m to get 4.25m without knowing the relationship between meters and centimetres. In oral arithmetic we would usually not refer to it as ‘two point five meters’ plus ‘one point seven five meters’ bin say ‘two and a half meters’ have to be added to ‘one metre seventy five centimetres’. Thus whereas written procedures are more general and normally distanced from meaning, oral procedures are more specific and help preserve meaning.

In example 3 we have seen the negative consequences of the distance from meaning created by written practices. This has been an area of deep concern for mathematics educators. Many studies have found that even when people carry out written computations properly, and demonstrate an ability to deal with a large range of numbers, they often fail to interpret their results correctly. A study called the National Assessment of Educational Progress was carried out on a large sample of school students, and it was found that even when school algorithms were adequately learned, students end up giving meaningless answers to arithmetic problems. One problem was: “An army bus holds 36 soldiers. If 1128 soldiers are being taken to the training college how many buses are needed?”. Out of 45,000 students aged 13 years, 70% performed the long division correctly. However, when asked to state the answer more than 13°000 students (almost 30%) answered that the number of buses needed was ‘31’ remainder ‘12’! Another 8000 students (18%) gave “31” as the answer.

(From the paper by: Carpenter, Lindquist, Matthews and Silver (1983), “Results of the third NAEP mathematics assessment,” *Mathematics Teacher,* vol. 76, p. 652-659)

Farmers versus Students:
A Brazilian Study

An interesting study was carried out in Brazil to contrast ‘street’ and school mathematics and understand how meaning was preserved or lost while solving simple problems. 15 farmers and 60 school students from class five and seven were carefully interviewed and observed. The problems were developed by first finding out how farmers used arithmetic in their daily work, and how they normally carried out measurements of land, volume, distances etc. We shall present two of those problems which deal with decimals.

Question 1

A farmer was going to build a gate. He had to cut a piece of wire 7m. long into pieces 1.5m each to fit the gate. How many pieces would he get?

A reasonable range for the expected answers was estimated. In this case, the range was 1 to 7 pieces. If no cuts are made there is 1 piece; if people simply ignore the decimal point they will cut 7m into 1m pieces and get 7 pieces.

Question 2

Suppose this is a piece of land 60m x 30m. (showing a rectangle drawn to scale). When you plant tea bushes, the space between bushes has to be 3 by 4 metres. How many bushes will be planted in this plot?

The results showed that farmers had a preference for oral arithmetic (though they knew how to write) while students- used written procedures. In Question 1, 90% farmers gave answers within the reasonable range, while only about 60% of the fifth grade students managed to do so. In fact, students gave some extreme responses which varied from 0.4 to 413 pieces! They had obtained these while trying to divide 7 by 1.5, because they did not know the algorithm to apply and also where to place the decimal.

An analysis of Question 2 also showed that the farmers always stayed close to the meaning of the problem, and therefore did not stray while interpreting the result. Almost all farmers who gave a final result adopted meaningful strategies while less than half of the students did so. An example of the way a farmer solved the problem:

"There will be fifteen bushes per row.
Because in each four metres you plant one bush. Then ten bushes will take forty metres, but there are still twenty metres to plant. Then you need five more bushes. It is four times five, twenty. Thus it is fifteen bushes per row.
Then there are thirty metres on the side. Thirty by three is ten rows. Ten rows in front by fifteen on the side. That's one fifty. Then it's right, it makes one hundred fifty bushes."

(taken from the book *'Street Mathematics and School Mathematics'* by Nunes, Schliemann and Carraher, Cambridge University Press, 1993)

Chapter 4

Local Knowledge and Folk Mathematics

One big tree
It has twelve branches
on each branch there are thirty leaves
of which fifteen are black
and fifteen are white- what are they?

It is impossible to count appa's money
it is impossible to fold amma's sari
What are they?
(translated from Tamil)

> Using a poetic riddle, with a rhythmic play of words, to tease and challenge the minds if learners, is indeed a potent teaching tool used by traditional oral societies to develop creative thinking and imagination.

These timeless oral riddles (from Tamil Nadu) about time, stars or the sky, lyrically reveal the nature of observations made by a society sensitive towards its surroundings. They also show how an oral society has managed to codify its knowledge for its young learners in an eminently effective way. Using a poetic riddle, with a rhythmic play of words, to tease and challenge the minds of learners, is indeed a potent teaching tool used by traditional oral societies to develop creative thinking and imagination. Even today numerous such gems, amazing in their philosophical, lyrical or even mathematical content, can still be found. Ironically, we the urban 'educationists' who plan and impose our curriculum on these so called 'illiterate' rural learners, have long last our own cultural moorings. Not only are we blissfully unaware of the traditions of local idioms and folk knowledge, but are also arrogant in our belief that these people can be treated as ignorant children while they are taught.

A cursory look at any adult primer shows how unimaginatively and mechanically we tend to 'teach' about the calendar, or about the notion of time, arithmetically divided into years, months, days, etc. We are often ourselves ignorant of the historical mathematical significance of the way calendars had been developed through various civilisations. Worse still, we suspiciously presume that all folk sayings would reflect their 'unscientific' bases, which we must somehow diligently guard against, from corrupting our own knowledge system. We therefore do not even try to establish a dialogue between our different systems of knowledge, and only lament 'their' inability to cope with 'our' norms of numeracy.

Some years back, in a modest attempt to understand the numeracy practices of present rural societies, one of us (L.S.S.) had undertaken an intensive study in seven villages of Tamil Nadu. Detailed observations, prolonged discussions and numerous interviews with the villagers had helped in gathering a lot of information regarding their ways of counting, sorting, measuring, etc. The study has helped provide many insights into how we could redesign teaching and learning methodologies and materials for adult education programmes. It is with such an objective that this chapter has been written, and is not meant to enumerate 'ethnic' traditional practices purely from an academic perspective. We are sharing this with the hope that persons engaged in the TL or PL campaigns would find it useful and would be motivated to explore similar numeracy practices of the learners in their local regions.

Telling Time

Almost half the villagers still used the positions of the sun (its shadow), the moon and the stars to keep track of time. Another quarter reckoned time through daily routine sounds, such as, the temple bell, the factory siren, the school bell, the sound of a given train, the time the mail arrived, or even the time of the mid day meal served to children.

About ten per cent seemed to make use of biological processes that depended on identifiable natural clock rhythms, such as, the crowing of the cock, birds chirping in the morning or m the evening while moving out from, or returning to their nests, or the coming home of cattle after a day's grazing. In fact, this last 'time-event' has a beautiful word in Hindi, called 'godhuli', which literally means the dust raised by the cattle (as they all return together). Used as a word to denote a time just around sunset, it conjures a host of related visual images and also the sounds of such typical scenes from the tapestry of our village life.

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Talking to people about how they could tell the time of the day had revealed many ingenious methods. In one such method a piece of straw was used as a measuring device, and its shadow was used to tell time rather accurately. The whole procedure was recited by a villager in a four- line Tamil poem, which when translated reads:

Take a piece of hay
Divide it into 16 parts
Stretch part of it horizontally
(the extent to be decided by the length of the shadow of the vertical part)
Count the number of vertical parts
For the number of 'nazhigais' passed.
(After sunrise or noon as the case may be)

Notice how optimally short (and sweet!) this seems as compared to how we might normally express this same idea in a precise 'literate' or 'scientific' style:

A piece of straw, conveniently taken to be about four times the width of four fingers held together, is taken and folded into sixteen equal parts. (First folded into half, then again, and again.) This piece of hay with sixteen visible parts is then bent like a letter L and held on the ground, with the vertical position towards the sun. The shadow of the upright portion now fells on the horizontal section kept on the ground. The upright portion is adjusted such that the length of its shadow is equivalent to the length of the horizontal portion. When adjusted this way, the number of parts of the upright portion indicate the number of *'nazhigais'* (the traditional unit equivalent to roughly 24 minutes) that have passed after sunrise, if it is forenoon, or the number of *nazhigais*' that have elapsed after noon, if it happens to be afternoon.

To measure time durations people use metaphorical terms such as, 'the batting of an eyelid' or 'snapping one's fingers' etc., as was revealed by their responses to many questions posed. In the case of those actions or events that lasted longer, people tended to actually measure time much less often. In fact, events rather than the time seemed to be important to most people. For example, for those occupied in agriculture, it was not really the time that needed to be kept track off after the sowing was done. The plants tell them what to do when, such as the right time to start weeding, to apply fertiliser, to start harvesting, etc. Similarly, in telling their own ages people refer to events rather than the number of years.

Most people said they possessed a traditional calendar in their house to know the date for festivals, the auspicious/inauspicious day or even such time of the day, full moon day, etc. Some used the almanac to get horoscopes made, to fix wedding days, and also to predict the quantum of rain in the current year.

Directions

Another interesting feature was the widely prevalent knowledge of directions, eight of them, amongst most people covered by this study, as compared to much fewer in the northern states. This seems to have some correlation with the rituals which have survived, that depend on certain directions for even simple things like ‘where to place the first brick of a house being constructed’. East is considered very auspicious, perhaps being the direction from where the ‘life-supporting’ diety, the Sun, rises. North East is considered to be the direction auspicious for commencing any construction, or even for white washing the house. The South is. inauspicious so the family diety or the family stove is never kept facing south-When someone dies he is said to have gone ‘southwards’ *(‘thirku’).* Incidentally, fishermen, whose lives (and living) depend on the directions of winds, have terms for both water currents and winds blowing in all such eight directions.

Owing to their knowledge of directions the villagers seemed to have a relatively good mental map of their village. However, specific exercises in reading maps and making maps need to be designed for neo-literates to help develop these faculties further. With more emphasis on’ village mapping’ and participatory resource management, it would be crucial to enrich these skills. It has also been found that women tend to feel diffident about their mental maps, training them to read maps greatly enhances their confidence in being able to move about on their own.

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Counting and Sorting

In a predominantly oral culture, number names rather than number symbols are commonly used. The available literature on folk lore and the above mentioned study indicate that number names have traditionally been communicated through popular songs, games, riddles and stories, many of which are still in use. Verse and rhyme help in memorising long pieces of often complicated information, in the absence of written aids. Moreover, numbers have meaning in specific contexts and are used for comparisons which are not just functional, but also serve to codify the empirical observations of societies and, at times, even their philosophy of life.

Elephants and humans both share the age of 100 years
For both cows and bullocks the years are only 20
The he-buffalo lives for 30 the horse for 32 years
The goal’s life span is only 12, the dog’s of about 15 years
While the camel lives somewhat longer till he is 73.

(A poem from ‘*Kanakaadhikaararn’* - an ancient text on mathematics)

A beautiful riddle, which shows how a pair of bullocks and the farmer ploughing his field may be considered as one holistic unit, inseparable from each other, is also a simple lesson in number names and addition:

Three heads
Ten legs
Two tails
Six eyes
Two hands
Four horns.   What are they?

Five in a row and four spaces in between. Say, what is it?
(Five fingers on each hand)

Brothers there are five
Without the support of the eldest
The younger ones can barely survive.
What is it?
(Thumb and fingers)

Interestingly, this one is not just an enumeration of the objects involved, but is a deeper reflection of the special characteristic of homo sapiens which is considered to have played a significant role in the evolution of humans. This ability, which allows us to touch each finger separately to the opposable thumb, and therefore work with tools, is responsible for most human creation.

> Numbers have meaning in specific contexts and are used for comparisons which are not just functional, but also serve to codify the empirical observations of societies and, at times, even their philosophy of life.

The ease of counting with speed is achieved through convenient unit-sets, typical for each different product. Unit sets vary from naturally occurring bunches (bananas or coconuts), to counting on five stretched fingers (banana leaves, cow- dung cakes, etc.), the volume held by one hand (betel leaves, paddy seedlings), and bundles carried as a headload or on the shoulders (sugarcane, bamboo). In many cases specific-sized containers, such as nets (with different sized 'eyes' to keep fruits of different sizes from falling out), baskets (tea leaves) or sacks are also used…

Multiples of such unit set are used for the counting of larger sets. For example, a decimal relationship exists between the following local measures, (each term has a meaning which can be concretely visualsed)

1 handful (pidi)	= 1 knot *(mudi)*	= 10 seedlings.
10 handfuls	= 1 cone *(kalasam)*	= 100 seedlings
100 handfuls	= 1 bundle *(kattui)*	= 1000 seedlings

Ingenious ways of visual counting are routinely used. Beedis, for instance, are placed in a regular bunch held between the tip of the forefinger and the thumb, so that the arrangement always contains 25 of them - 3 in the middle, 8 around them in one circle, and 14 in the second concentric circle, making a total of 25 in the bundle (kattu). 40 such bundles make 1000.

Recordinng of counts is done through many different methods and most local games provide some insights into this. For instance, games played with tamarind seeds (described in some detail in chapter 7 on 'Basic Arithmetic and Applications'), or with cowrie shells and pebbles, indicate the ways in which counts are recorded and keenly kept track of. The excitement of winning or losing keeps people fully engaged, and also is a motivation to count and add or subtract speedily as well as correctly.

Estimation

Since most counting and sorting is done by participating in specific production processes, people acquire the ability to make very accurate estimations as part of their working lives. Estimations are the basis for all measurements, and there is normally a consensus as to which type of unit is to be used for measuring what and when. Even though people use a variety of units for different situations, they do not necessarily know or need to convert from one type to another. This may he true of most of us too, who cannot easily estimate heights of persons in metres and centimetres (having been accustomed to only feet and inches) though for cloth or distances we have been using the standard metric units. However, learning to inter relates various types of practical units in different situations can enhance the ability and confidence of adult learners.

An interesting poem illustrates an unimaginably wide range of length measures, created through a rich life-world of imagery and experience. Unfortunately, the English translation cannot possibly match the sense of rhyme and play of words in the original. It is clear that on the small end of the length range this scale is only suggestive, giving rough estimates (not exact relationships) though it becomes closer to realistic measures on the larger end of the spectrum.

In addition, there is a measure here called *'koopidu dooram*', .which literally means 'calling distance', and suggests that there was folk knowledge of the fact that sound travels only a finite distance! This may be a good lesson even for some of us who teach school children, to introduce such scientific concepts more naturally and poetically.

8 atoms	= 1 speck in the sun's ray
8 specks in the sun's ray	= 1 speck of cotton dust
1 cotton speck	= 1 hair point tip
8 hair tips	= 1 small sand particle
8 sand particles	= 1 small mustard seed
8 small mustards	= 1 sesame seed
8 sesame seeds	= 1 paddy seed
8 paddy seeds	= 1 finger width
12 finger widths	= 1 span
2 spans	= 1 cubit
12 cubit	= 1 stick (kol)
500 'kols'	= 1 'Koopidu Dooram' ('calling distance')
4 'koopidu doorams'	= 1 'kaadam' (roughly 1.2 kms)
4 'kaadams'	= 1 'yochanai'

People engaged in different vocations dearly have their own measure for measuring lengths of different objects

Linear Measurement

People engaged in different vocations clearly have their own measures for measuring lengths of different objects, such as, mats, bails of cotton, rope, etc. Linear measurement involves the process of recognising the linear dimensions of any object, describing the-same in terms of some known thing in the environment, and making a choice of a convenient unit common to all in the community. This is followed by making an estimate or actually measuring the linear dimensions of the object. There seem to be roughly four stages in the way this is actually done.

Stage 1

Recognition of linear dimension - a tall tree, a short person, a long distance, sparse rain, etc.

Stage 2

Comparison with a known object

• the depth of water in terms of the number of steps immersed within the water, the rope length that gets wet, time taken for a stone to reach the bottom of the source, etc.

• distance in terms of number of houses on the way

• size of the mouth of a cooking stove or 'chulha' in terms of the size of the vessel placed on it, or the number of firewood pieces to be placed in it.

• arm girth and hip girth in terms of specific sized bangles, belt, or petticoat

• diameter of a hole in the strainer ladle in terms of known objects, such as, a mustard seed, red gram dal, a pepper, a black gram dal, the pupil of the eye, etc.

• rainfall measured in terms of levels of lakes and lanks, or the level of water collected in certain vessels or the grinding stone kept out in the rain, the flow of water, or the quantity sufficient to plough land.

Stage 3

Choice of a unit for measurement- usually in terms of a stick, siring, or body parts.

Stage 4

Recognition of the need for and use of standard units and tools for more accurate measurements.

The Imperial and Metric units are both used simultaneously and common tools are found to be the wooden scale or a measuring tape.

The demands and descriptions of linear measures are such that any individual or group could be at any of these stages at any given time. A better understanding of measures results not from merely moving from stage 1 to stage 4, but from an understanding of the interrelatedness of these stages and skills in using them with ease. Numeracy programmes should therefore ensure that learners appreciate the process of evolution in linear measurement and acquire a facility to move from the traditional measures to standard measures, wherever required. Activities need to be organised to help learners recognise the different stages of the process - from estimating or 'not measuring', to seeing the linear dimension in an object with reference to another, to choice of a unit which could vary with individuals (non standard), and finally to choosing a unit common to all. in and outside the village (standard unit).

Capacity and Weight Measures

It is found that capacity measures are more prevalent than those for weight since they happen to be more convenient and they require no weighing instrument. Agricultural produce, provisions, manures and fertilisers, etc. are mostly measured in traditional units, with their own conversion tables. Metric units are generally used only in shops. However, even after rice and grains are bought in shops they

are measured at home in traditional storage containers called ‘padi’ or ‘ollocks’. One kilogram office is considered equivalent
to 5 ollocks or a tin of a specific size. Similarly, equivalences of litre measures for milk, buttermilk or oil arc normally known in terms of specific containers used at home. One ladle is meant to hold 10 spoonfuls.

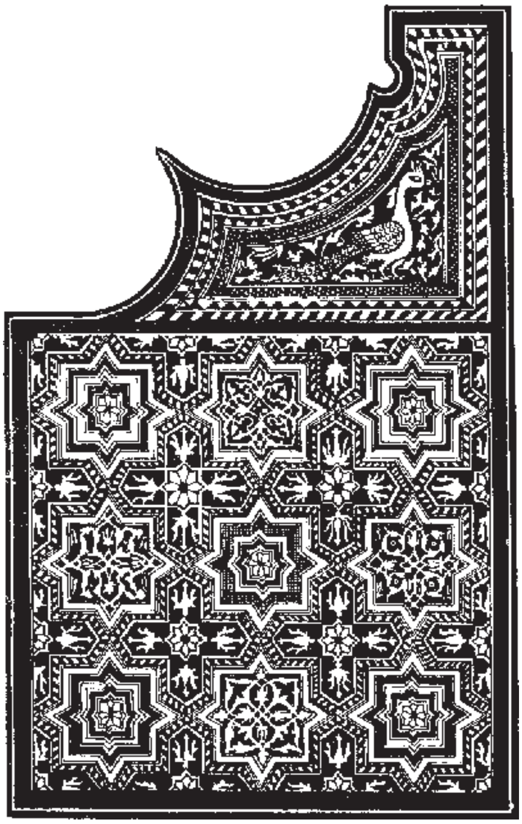
Extensive use of body measures is still found to be made and these are discussed in Chapter 6 on ‘Measurement’.

Fractions

A very interesting observation made during our studies is that in many states of South India very intricate fractions still exist in everyday vocabulary. (This seems almost like the markedly more intricate sound patterns for the *‘mridangam’ or’pakhavj’,* and other percussion instruments used in Carnatic music). For instance, in Tamil and in Malayalam,

1/2	‘ara’
1/4	‘kaal’
(1/2)x 1/4	‘araikaal’
3x1/4	‘mukaal (‘moon kaal’ where ‘moon’ stands for 3)
(3/4) x 1/8	‘mukaal arakaal’

This is however not found in the northern states. It would be useful for resource persons working in different literacy campaigns to look at these details in their own respective areas and compare notes. Primers must introduce fractions and division by suitably referring to such rich terms and concepts already existing in those cultures.



Details of a mosaic of ivory inlay, comprising stylized floral, leaf, star and geometrical patterns, with a peacock and birds, from an ebony door Amber Palace, Rajasthan. Early I7th century A D. Notice, how this pattern also forms, a “tessellation“ or a repeated tiling pattern discussed on the next page.

Art, Culture and Folk Mathematics

The famous mathematician Hermann Weyl once wrote:

'One can hardly overestimate the depth of geometric imagination and inventiveness reflected in these patterns. Their construction is far from being mathematically trivial. The art of ornament contains in implicit form the oldest piece of higher mathematics known to us".

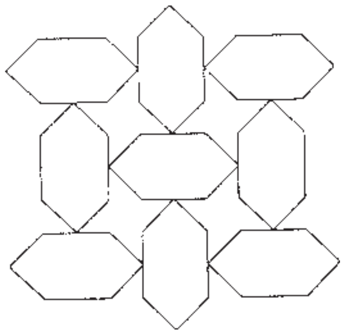
This passage was written with reference to an important theorem in higher mathematics, which states that "there are exactly 17 two-dimensional crystallographic groups" and popularly implies that 'there are exactly 17 different wall patterns'. If we look at traditional designs found on floors, fabrics or walls, it seems that many centuries back people in different cultures had knowledge of all these 17 different patterns.

Examples of floor patterns from early Egyptian, Greek and Chinese civilizations.

One of the purposes for studying geometry is to discover order and pattern in the universe. People could be asked to find their own tessellations, or 'special space filling' patterns.

In fact, a famous artist Maurits Escher (1898-1972) whose art is a source of inspiration to many mathematicians, had studied patterns made on the walls of a place called Alhambra in Spam, About those special patterns he says in his book:

"This is the richest source of inspiration I have ever stuck. A surface can be regularly divided into, or filled up with, similar shaped figures which are contiguous to one another, without leaving any open spaces."



A floor tiling from Taj Mahal

These patterns are called 'tessellations' in mathematics. But the term should not make it seem to be a difficult idea - after all such patterns can be made even by non-literate persons and they provide an enjoyable activity. People could be asked to find their own tessellations, or 'special space filling' patterns. If our non-literate women have, over the centuries, discovered hundreds of thousands of 'kolam' patterns and even made a festival out of the activity, why should we not consciously introduce more such mathematically rich ideas into our TLC culture. We only hope our literacy or rather 'numeracy' campaigns would become more alive to the influence of art and culture on people's lives (and naturally their minds too!). Mathematics is closely linked to our music, art, poetry, games, as well as our ways of rational (and irrational) reasoning. We must only open our own minds to explore ways of devising teaching learning materials and methodologies which creatively build on such natural linkages.

If our non-literate women have, over the centuries, discovered hundreds of thousands of 'kolam' patterns and even made a festival out of the activity, should we not consciously introduce more such mathematically rich ideas into our TLC culture?

Certain odd shaped figures that one wouldn't expect to tessellate the plane, interesting tessellations

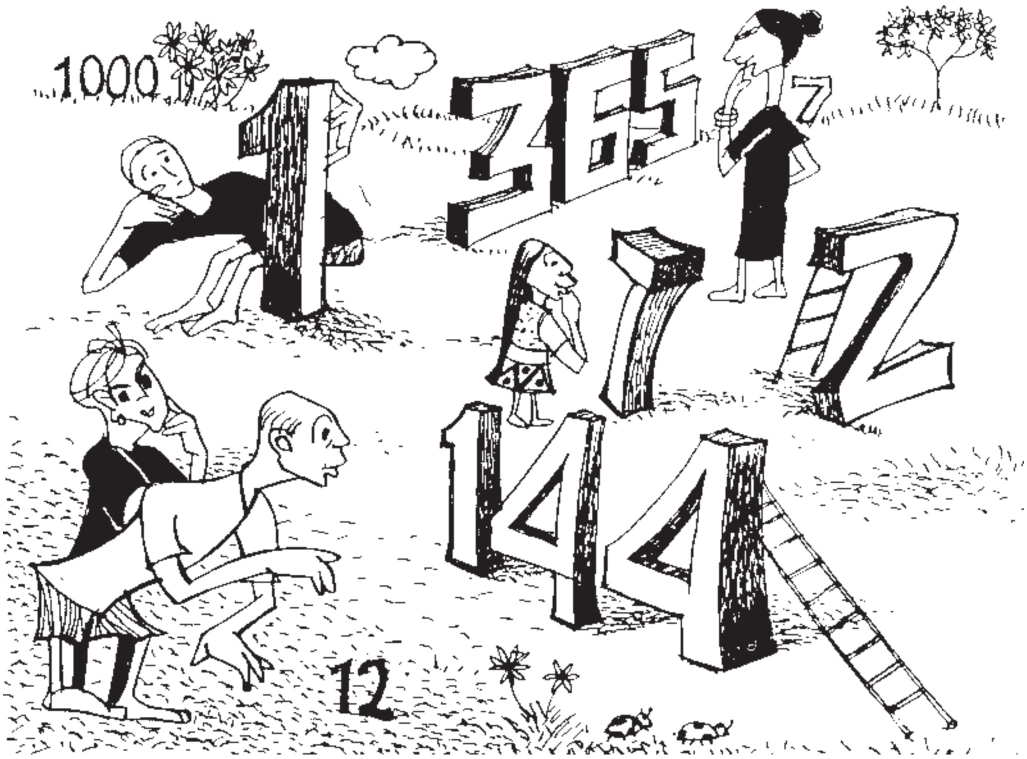
A drawing by M.C. Etcher

Chapter 5

The Meaning of Numbers

As Ramanujam lay dying in hospital (in England) the famous mathematician G.H. Hardy regularly used to visit him. One day on entering his room, Hardy remarked, even without a greeting, 'The number of my taxi-cab was 1729. It seemed to mea rather dull number." To which Ramanujam replied "No, Hardy! No, Hardy! It is a very interesting number. It is the smallest number expressible as the sum of two cubes in *two* different ways." (C.P. Snow, *Varieties of Men,* Macmillan 1968, p. 32)

$1729 = 10^3 + 9^3 = (10 \text{ x}10 \text{ x } 10)$
$1729 = 1^3 + 12^3 = (1 \text{ x } 1 \text{ x } 1) + (12 \text{ x } 12 \text{ x } 12)$



In teaching literacy skills, we introduce letters through meaningful words. A similar approach is worthwhile with numbers as well. But what is the 'meaning' of numbers?

How many stars on your lehenga?

In teaching literacy skills, we follow the methodology of introducing letters through meaningful words. A similar approach is worthwhile in the context of numbers as well. But what is the 'meaning' of numbers?

The teacher should maintain a number diary for her centre, which is basically a database of numbers and their meanings, arising out of discussions in class. Often learners think that they do not have much to do with numbers. The VT could therefore start by making a simple 'number statement' about herself or himself; for example: "I weigh 52 kilograms" "My uncle has only 9 toes" "There are 7 people in my family" "I travelled 11 kilometres to reach the main market"

The group is then encouraged to carry on the process. Each person is expected to state her own number fact. The VT initiates by a series of suggestions of the kind given below.

The number of persons in your house?
Your age?
The number of films that you have seen?
The number of mangoes you expect to buy for 10 Rupees?
The number of the bus you take to go to .... from….?
The cost of one kilo of wheat you bought?
The number of stars on your 'lehenga' (skirt)?
The number of buckets of water you use every day?
The number of trees in your village?

Answers to these questions are in numbers, and these numbers as well as their significance is to be recorded in the number diary. These are the meanings that numbers possess in the minds of learners (and the teacher), and for most (if not all) of the exercises one must use only these numbers. The idea is that when an addition or multiplication problem is given later on, if the learners seem baffled, an interpretation can be quickly provided in terms of the meanings recorded earlier: Moreover, this exercise motivates learners about how they keep encountering numbers in life.

Another way of continuing this exercise is for the teacher to tell a number as a quantity of some object aloud and for the learners to respond with some objects of the appropriate quantity. For instance, the teacher says '100' and a learner responds with '100 jasmine flowers' ; then the teacher says '50' to which another learner responds, '50 rupees', and so on.

When this is understood by all, the learners can be encouraged to say aloud, in turn, a number as a quantity of some object. For instance, the round may proceed as: "10 fingers", "2 eyes", "50 paise", "8 drumsticks", "15 onions" etc.

Learners must also be encouraged to ask questions (of the type listed above) to each other, to look consciously for numbers and to report in the next class, and thus contribute to the class database of numbers.

How Numbers Grow

It is likely that learners typically think of only small numbers as meaningful in everyday life. They must also be encouraged to think of large numbers, both in order to motivate the study of arithmetic and to get a feel for numbers encountered later. For this, questions can be posed as follows :

'19 hairs!'

How many mangoes does a tree yield in a year'?
How many days are there in a year?
How many leaves does a typical mango tree have?
How many chapatties do you make in a year? How many of them do you eat?
How many stars are there in the sky?
How many people live in our village?
How many hairs do you have on your head?

It is important that learners try to guess and estimate answers to these questions. Do not accept any response of the "I do not know" or "No idea" variety. The main idea is not to arrive at exact numbers as answers to such questions but to make estimates so as to decide whether the answer is in hundreds, thousands or lens of thousands etc.These words (like 'thousand') are themselves usually familiar to learners, and a discussion on whether the number of leaves in a neem tree is in hundreds or thousands can be very lively.
(Also see 'A Feel for Large Numbers' in Chapters 7 and 8)

Experience with numerous discussions shows that learners do understand that there is no limit to the growth of numbers, though they do not articulate it in such terms.

> The main idea is not to arrive at exact answers to such questions but to make estimates so as to decide whether the answer is in hundreds, thousands or tens of thousands...

> A mathematician, like a painter or poet, is a maker of patterns. G. H. Hardy, A *Mathematician's Apology*. Cambridge University Press, 1969.

A good game is to challenge learners: "whatever number you tell me, I will tell you a bigger number than that". Initially the bids are small, like "40 etc." when the response is also slightly bigger. Gradually, when numbers get to thousands, the teacher should go for much larger ones. For instance, when learners come up with "5 thousand" the teacher should respond with "10 thousand" and not "6 thousand". This game is worth it only if at least half the class know words for thousand, lakhs etc. (They need not know that hundred thousands make a lakh, but should know that a lakh is "much bigger than" a thousand, that is enough).

One can similarly work towards a fed for very small quantities as well. Fold a paper into half, tear off, repeal and work on with "halt of half of...." till you can't go further. Simply point out that there are names for quantities got at each stage, and that the names never stop. However, this can place a strain on the learner's fragile understanding, and is not worth pursuing further unless learners show case with it.

> Looking for regularity is a learning skill in itself. Numeracy work books must contain a variety of exercises which focus the learner's attention on patterns and shapes.

Patterns and Shapes

'Numeracy work books must contain a variety of exercises which focus the learner's attention on patterns and shapes These are not to be bunched into one or two "chapters' but interspersed throughout the text They are an essential component of numeracy and form the background for numerical operations Looking for regularity in given objects/pictures is a learning skill in itself and provides a basic training in science (Also see 'Art, Culture and Mathematics in Chapter4) Exercises in this regard can be us follows:

• *Similarities and differences.* Ask learners to collect different kinds of leaves, draw their shapes on paper and discuss the similarities and differences.

• *Pick the odd one out.* Give a sequence of pictures, and ask the learner to pick the "odd' one out: there is plenty of room for imagination here ..... the odd one may differ from the rest due to kind (rest are tools and this is not), size (others are big, this is small), shape (rest are circular, this is elliptical), number (has four more dots than the rest), and so on.

• *Identification of extremes:* Given a sequence of pictures, ask learners to identify the largest and the smallest, the tallest and the shortest, the biggest and the smallest, and so on. Encourage them to not only point out these, but also refer to them by ordinals ("the third pigeon is the biggest and the fifth one is the smallest," and so on).

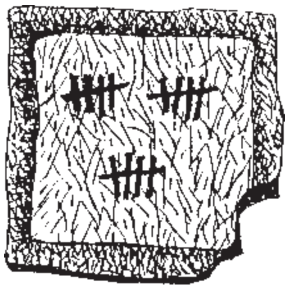
Encourage learners to draw kolam patterns, enlarge or reduce them and predict how many dots would be required for a design.

• *Sorting exercises:* Ask learners to identify the pattern in a given sequence of pictures and re-order them according to size, shape, number or whatever.

• *Continuing the sequence:* Given a sequence (of pictures, shapes, numbers), ask the learners to fill in the gaps or to continue the sequence by guessing the next entries.

• *Tangrams:* The ancient Chinese pastime of creating different shapes and figures trom only seven pieces cut out of a square, (see page 53)

• *Rangoli or kolam patterns:* Many learners would be familiar with rangoli or kolams (patterns made on the ground, in South India). Encourage tnem to draw a variety of these and show the regularities in them to other learners. Also ask them to enlarge or reduce a kolam pattern and predict how many dots would be required for the new design.



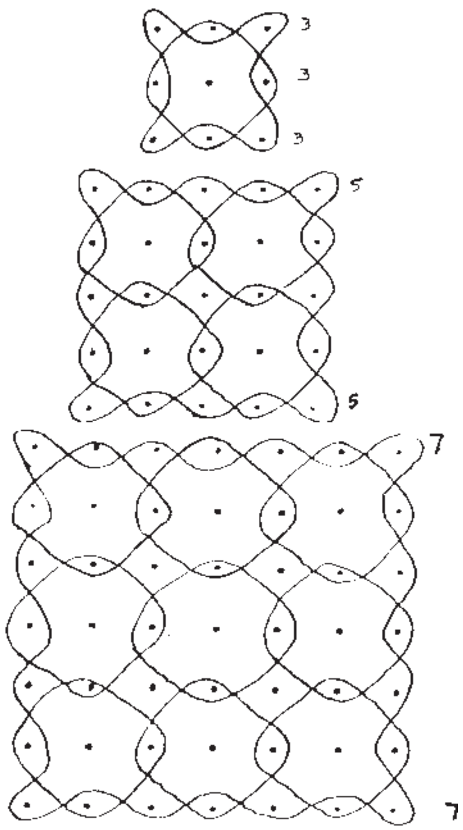
• *Abstract patterns and shapes:* It is important thai the sequences of pictures referred to above should contain not only concrete objects (flowers, leaves, birds etc.) but also abstract patterns (dots, triangles, etc.).

• *Counting by regrouping:* The patterns presented should encourage the learners to start counting by regrouping: for instance, it is easy to see, when presented with pictures of 5 mangoes and four, that the former has one more than the latter. This becomes harder to do 'by sight' in the case of 15 and 14, and impossible in the case of 50 and 49. Learners would also see that it is advantageous to have objects arranged in a specific order rather than given as a random bunch.

Folk-art of Kolam

Kolam is the most popular of the visual folk-arts of Tamil Nadu. This is believed to be 5000 years old! This art is done generally on the floor at the place of worship or the main entrance to the house. The designs are made with admirable ease. No tools are used. The ingredients used are rice flour or powdered quartz (a kind of white stone). Hence it is generally white in colour. The flour is taken between the thumb and the forefinger and dots are made before the lines are drawn. The designs are produced using the grid of dots as bases. On special occasions different colour materials are used. They are dry colour powders produced from soils, leaves, charcoal, plant roots, burnt earth, bark of trees, and coloured stones available locally.

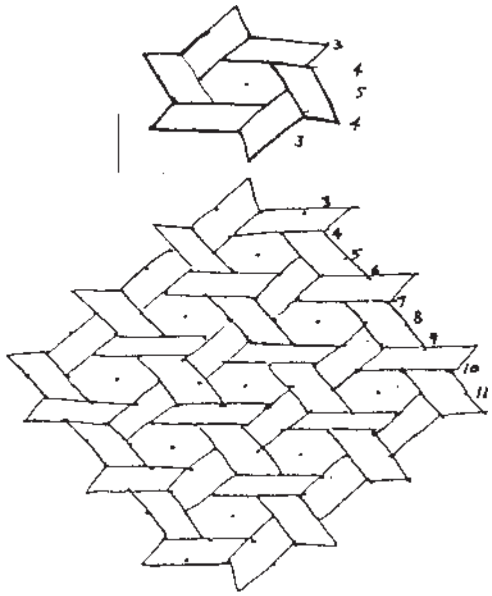
These designs are handed down from one generation to the next through the process of socialisation. These designs are generally made by women. Young girls start learning this art through observing older women practising it everyday. Every year during the period from mid December to mid January, for a whole month, the whole frontyard of the house is filled with fresh kolam designs everyday. This is done early in the morning after sweeping and cleaning the place with cow-dung water. During this month several women join together and collaborate lo make these designs. Many take this opportunity to study and learn different designs, stopping at places with attractive kolams, counting clots and looking at me connecting lines. In other words, kolam as an art is the focus of most activities during this month. On festive occasions, especially those connected with temples, many women jointly undertake the responsibility of filling the vast floor space with kolam designs.



A kolam around dots- an exercise in enlarging the basic pattern

The kolams are mainly of two types: single- unending line kolams and multiple- line kolams. There are kolams in which lines are drawn around the dots and others in which dots are joined. By proportionately increasing the dots a simple design can be enlarged.

The movements made at the time of designing kolams are considered to be natural movements. Arithmetical operations of enumeration and computation of addition, subtraction, division and multiplication are possible while counting dots, rows of dots — equal number and unequal number. Besides, other mathematical abilities of enlarging designs sustaining the same proportions, adjusting lo uniformity and symmetry, and widening the visual field could be developed using this art. Cognitive skill of understanding spatial relations is in-built into this art. It is amazing that with a set of five dots in a row and five rows of five dots, one can produce one lakh or one hundred thousand different designs. There is immense potential for creative expression in this art and that is one reason it is still alive. Each kolam design has a specific name that is meaningful to the people, and hence is easily identifiable by them.



A kolam joining dots

Counting in Sequence

Rather than ‘teaching numbers’ the teacher should first check whether learners can already count up to 100. If they know the sequence in words, the teacher should check that they know the meaning of the number words as well. It may well be that a learner can count “forty one, forty two................” and soon up to fifty, but is still unable to tell which is greater, 43 or 52. In such an instance, it is important to consult the number diary and turn the question into a contextual one: which has more mangoes, a tree with fifty two mangoes, or one with forty three mangoes?

Local Games

A number of games abound in every region of India, which involve counting in sequence, and the learner should play these games in class. A good example is snakes and ladders: the layout of the board on which the game is played, already gives the learner a good and useful mental picture.

Moreover, on every move, the learner throws dice and moves by the thrown amount, which helps, in counting. When there is a ladder or a snake close-by the learner is tensely awaiting the dice outcome praying either for or against a specific number, and this is again a useful arithmetical exercise.

Other useful games of this type are backgammon, pallanguzhi (in Tamil Nadu) etc. Local games should be identified and used for this purpose. One popular 'seed' game played in Tamil Nadu is described below:

Blowing Tamarind Seeds

Two or more can play this game. Children as well as adults play. Those who play bring as many seeds as they can and pile them up in the centre. Each player takes turns to blow the pile three times and pick up the seeds one by one without touching or disturbing the seeds that are closer to the one being picked up. If a neighbouring seed shakes while picking up, the player loses his turn. While blowing the seeds the mouth should not touch the pile.

The force of breath employed in blowing the pile for the seeds to get widely scattered, thus facilitating picking up a large number of seeds by the players, make it an enjoyable and absorbing game.

Counting of piles of seeds, comparing with others' catch, getting enthused to scatter more seeds next turn, are observed while the game is in play.

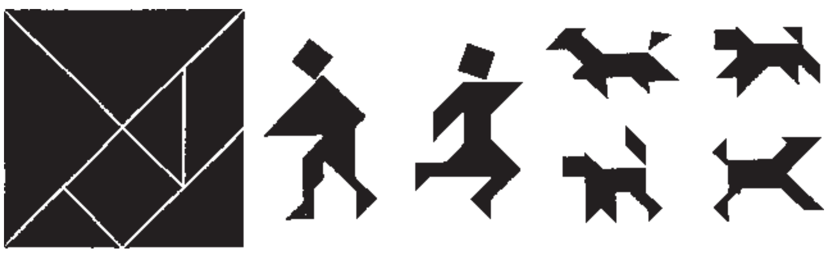
In seasons when the tamarind seeds are available in plenty every house has a gathering for playing this game. Though children play this game in large numbers, adults do play it too, and enjoy it.

Assessing Basic Counting Skills

Since there are only ten basic symbols, teaching learners to write numbers is not very difficult. Nevertheless, it is important to let learners achieve number writing at their own pace. Writing comes only after it is ensured that learners are at ease with verbal counting.
The teacher can be confident that a learner has acquired basic counting skills if –

- Given a quantity of seeds (stones, pebbles,.......) the learner can count and tell how many there are.
- Given two such quantities, the learner can, if necessary, count and check which has more.
- The learner can recite numbers in sequence.
- Given a set of numbers, the learner can tell which is the smallest or biggest among them, and can sort them in ascending/ descending order.
- Given a sequence of numbers, the learner can fill in gaps, in the sequence or continue the sequence.

While numbers up to 100 do include two-digit numbers, the concept *of place value* (units and tens) need not be understood at this stage, and can wait for later.



Chapter 6

Measurement

A depiction of a time measure of a journey that has taken two days, two nights and half a day, shown by two suns, two moons and half a sun on the top of the panel.

Traditional Units

Measurement of time, length, weight and volume constitute the most important manner in which quantitative thinking informs our daily lives. Most people educated in the modern idiom are conversant with the standard measures used for these attributes and are familiar with at least the following units :

Time : minutes, hours, days
Length : metres, centimetres, kilometres
Weight: grams, kilograms
Volume: milli-litres, litres

The British legacy of yards, pounds and ounces does persist in usage, but the metric system has generally managed to permeate public discourse in India.

As observed earlier, one of the stated objective of numeracy instruction to adult learners in TLCs is to make them conversant with the metric system. On the other hand, the Indian people have traditionally followed several systems of measures of their own, and Indian languages entertain a rich vocabulary of measure terms. Starting *ab initio* with teaching the metric system, entirely ignoring the learners' existing knowledge and competence, is unwise and almost all TLCs have done just that. Hence the first important maxim is: identify learners' existing methods of measurement and their vocabulary. All discussions on measurement should build on this vocabulary.

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The acknowledgement of traditional terms to be as valid as the modern ones is not merely desirable but essential. Building on this, the second step is then to motivate learners towards the need for standard measures and the necessity of learning these measures. Unless the learners' consent is obtained, standardization is merely an imposition of an external will on learners. The fact that commercial transactions are increasingly in terms of metric units is to be pointed out. This implies that the third step is to point out situations in daily life where learners have already encountered terms like 'metre', 'kilograms', 'litres' etc...

This establishes the modern terms to be meaningful, and initiates the process of translation to standard units. The fourth step, namely establishing metric equivalents of traditional terms must begin with situational usage rather than conversion tables. For example, in Tamil, we should begin with equivalents .for 6 *gajams* of sari (5.5 metres) and for 2 *aazhakkus* of rice (400 grams), (It should also be noted that these equivalences are approximate but functionally adequate.)

Only after passing these several stages, we get to the discussion of the metric units themselves, introduce the terminology of these units and study the rational structure of these units. In particular, the fact that they occur in multiples of ten makes the structure easy to understand. The conversion table, both from the traditional units to metric ones, and the other way, can then be studied in detail.

Many a Measure

Quite apart from the issue of traditional and modem units of measurement, it should also be emphasized that there is nothing like *'THE system of units'* when it comes to use in everyday life. We use a variety of units in daily life in mixed modes and are comfortable with them all. Below, we present examples of such usage.

(This section deliberately includes terms used as natural measures both in Hindi and Tamil. One can see that without knowing the exact meaning the reader can still get an idea of the process.)

Time

Kaia woke up as the cock was crowing. She folded up her mat in a jiffy, and went out to clean the frontyard. In a great hurry, she heated her *kanji.* When the water had come to a boil, she added *noi* and stirred it. When it was done, she took it off the stove. Soon she would have to leave for the fields, where she would have to work till noon. Her husband would have to leave in time so that he could be at the BDO's office on time. He had already been warned once, that officers would not wait for him. Would he make it to the eight o'clock bus today?

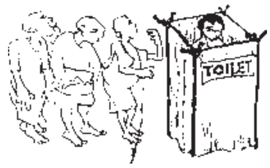
Length

Kodandam was so tall and thin that when he walked it was like a man on stills. With arms like drumsticks and legs like *laukis,* he was a walking cartoon, and this impression was further heightened by his nose, which was as long as a full piece of chalk. He walked about 20 km. every day, visiting at least 3 villages, to sell the dhoties he carried on his head. He usually sold more of the 4 *muzham* variety than the 8 *muzham* ones, though he wished it was the other way, as he got more profit on the latter.

In fact, the natural measure is usually contextual and provides an imagery which makes it easy to understand the process.

Weight and volume

Cut 4 large onions coarsely. Fry them well using 2 *karandis* of oil. Remember to fry two spoons of *jeera* in the oil before adding the onion. When browned, add a *pao* of coarsely chopped tomato to let it fry well. When the mixture begins to stick, add 2 *aazhakkus* of cleaned rice, and stir well. Add three tumblers of water, and close the vessel tight.



"This activity is suppose to lake minutes, not hours"

The point here is that there is an appropriate natural measure for every occasion. In fact, the natural measure is usually contextual and provides an imagery which makes it easy to understand the process. The standard measure, in such cases, could even sound awkward and incomprehensible. This may be emphasized in two ways in the primer:

• Giving such passages and asking learners to identify the different measures used.
• Converting all measures to standard units and raising a laugh at how ridiculous that sounds.

Which Measure to Use When

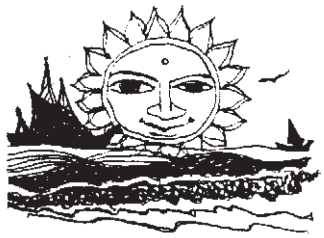
A second set of exercises is required to generate a discussion in class of which is the appropriate measure for each situation. The class thus builds up a table of the various situations that arise involving measurement in daily life, and the measures that are relevant in each. As an example, the learners list different items of food and other items used at home: rice for cooking, salt, *dal, aata,* tea, milk, kerosene, oil, tamarind, chilli powder etc.; items used for agricultural work, manure, grass for cow, etc. (One way of doing this is in the form of a game where these are written on chits of paper and each learner picks up one, thinks of the relevant measure and speaks out.)

The main point here is the need for discussions around the following questions: what are the different measures that we use and why do we use them? In what way can we relate them all?

Such an exercise needs to be carried out for all manners of measurement. Below are some example: (Most of the activities mentioned can be carried out as games on the lines suggested above; this is left implicit. Moreover, wherever learners are required to answer, they can do so in any units of their convenience.)

Time Measures

Learners are asked to list the amount of time it takes for various activities, which can be categorized under different orders of time.



• *Order of a few minutes*
Estimate the amount of time it takes for various activities, which typically take a few minutes: for example, brushing teeth, having a bath, eating a meal, etc.

Try also ambiguous ones like, ‘time taken for the sun to set’! How long does it take for the full disc to disappear below the horizon?

Order of a few hours
Estimate the time taken for activities which typically take a few hours: for *idli* dough to ferment, for curd to set, for my longest shadow to reduce to its shortest etc.

Order of a few days
Estimate the time taken for activities which typically occur on a natural scale of days, such as, for a raw banana to ripen, for a flesh wound to heal, for a chick to emerge out of an egg, etc.
Similarly we could estimate the duration of activities lasting weeks, months, years etc.

One process, different time orders
Now we attempt to describe durations on mixed scale mode. Let us look at the process of making an *idli* from the very beginning and describe its path in time. From the point of growing rice or *dal,* harvesting, storage time at the granary, soaking, grinding, fermentation, steaming, eating and (hopefully!) digesting.

Contrasting time orders
Attempt to estimate durations at either end of the spectrum, like those that involve just a few seconds, and those that may take several lifetimes.

Listing activities possible within a given time measure

A converse set of exercises, where the learner is given a range (say 5 to 10 minutes), and she lists a whole set of activities she can think of which typically lake that much time (at least 5 minutes and at most 10 minutes, in this example).

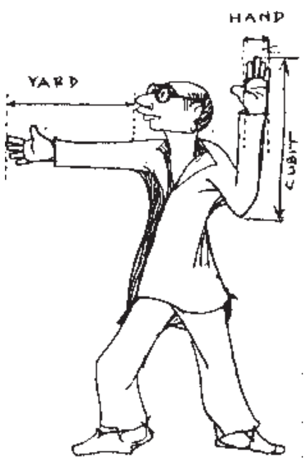
'Felt' time seems to pass slowly or remarkably fast, depending on events and the activities we are involved in. Language and folk literature have rich modes of expression for these experiences.

Relative time

The learners, being adults, would also appreciate discussions on relative time. It is a common experience that 'felt' time seems to pass slowly or remarkably fast, depending on events and the activities we are involved in. Language and folk literature have rich modes of expression for these experiences. Learners could be asked to list these.

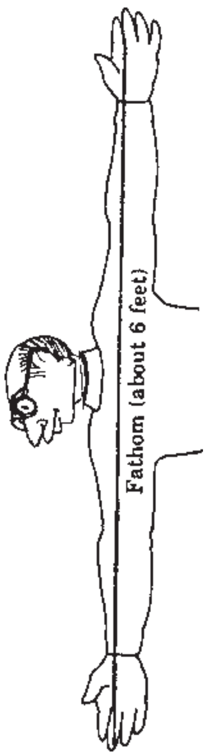
A similar set of exercises, involving the range of different measurements, can be done for other measures such as length, weight, volume etc.

Body Measures



Extensive use of body measures is important, particularly for *length.* In terms of exercises, the above progression is done with units like finger-width, finger-length, span, cubit, feet, fathom, etc. For depth, people often use units like ankle-deep, knee-deep, man-deep, etc. In addition, units like a *chatai*-length, a *dhoti*-length, length offish etc. are also useful.

When it comes to *volume* measures, similar units are available using *chittigais* or a pinch, in Hindi also called *chutki-bhar' (that* which can be held between thumb and forefinger), handfuls etc. Similarly, there are local terms for palmful, handfuls, and even stomachful (as in *'pait-bhar khana').* Interestingly, in Tamil, the finger measure *'sarangai'* is used to denote the amount scooped up by the fingers.



Examples of substances measured in such a manner.

- *Taken as a pinch:* Salt, snuff, herbs, medicinal powder, etc.
- *Scooped up by the fingers:* Mustard seeds, cumin seeds, haldi powder, etc.
- *A handful:* Flour, pulses, sugar for sweets, peanuts, flowers, soil, etc.
- *A palmful:* 'Prasad' during ritual prayers received with a cupped palm, etc.

Relating the Measures

Given the variety of measures in use, it is important to relate these, and an understanding of their rationale is an essential numeracy skill. This requires both comprehension of how within any system (traditional or modem), the bigger units are related to smaller units, and how a unit in one system relates to a unit in another system. The primer should have passages which refer to these relationships in both traditional units and in standard modern units. Below are examples of such passages, but you would need to construct more according to your own local units.

1. At half past six, the sun rose and the valley was filled with the glow. They got up and washed. It took them nearly forty five minutes to get ready and start moving. They had another 8 kilometres to go before they reached the foothills, and they usually took about 10 minutes to cover a kilometre. In the afternoon, progress would be slower. It was quite hot when they started climbing. This being a day in May, the rocks powdered to dust under their feet. At noon they were only halfway up, now barely crawling ten fathoms in ten minutes.

2. She dreamed of a house which would be her own. She would not have to crouch like this in her house, the roof would be al least 4 metres tall. There would be a big kitchen in a separate room: it would be a space 4 metres long and 3 metres wide, all to herself. Every wall in the house would have windows halfway up. The windows would have nice grilled patterns; she would want smaller and more intricate patterns than the ones in Jamila's house, which were merely squares 8 centimetres wide. The door would be wide and on festivals she would decorate n with a string of flowers four *muzhams* long.

3. He looked about the water tank he had to clean. "I wish my hut was this big", bethought. It could hold 5 rows of men like him, ten deep to a row. He must have bailed out nearly 30 litres of water just to clean the tank. 'He wondered how many litres of water each of the 20 families in the building must be using each day. They said the tank emptied every day. He thought of the building as a huge person drinking up so much water in a day; how big would such a person be? How many times bigger than himself?!

4. They had to decide now; could they afford to buy five kilos of chicken? The Ramaiya family were known to be gluttons at dinner. But then if their daughter was to be married into the family, there was no point in worrying about this. They decided also to buy 200 grams of *khoa,* which would be only for the guests. Even 50 grams of cashewnut would have helped greatly for the sweet, but that would have to be skipped. They would make do with a *payasam (kheer* or pudding) for which they could buy one *pao* of special palm *gur.*

At this stage, the class can construct tables showing the relationships between known traditional units: for instance, a fathom is 4 cubits long; 4 *pao's* make one *sair* and so on. This would motivate definitions of standard measures like l metre = 100 centimetres, which are to come later.

Standard Measures

It is important to discuss with learners the relative merits and. demerits of using body measures in different situations. Importantly, the flexibility of this system and ease of estimation needs emphasis. On the other hand, the associated inexactness and variation from person to person often makes commercial transactions tricky. Would either a seller or a buyer be happy with quantities like a handful of *elaichi* (cardamoms)? Or a pinch of gold?!

The standard units are introduced systematically at this stage. For time, clock time and calendars are studied. It is best to make a cardboard clock and a calendar for the current year in the class itself. For the other measures, the standard measures (ruler, metre tape, measuring glass, kilo weights) can be borrowed for measurement activities in class.

The fact that learners typically use their own "domestic standard" measures needs to be discussed. For instance, a vessel at home serves as a benchmark, and it is known that a litre of milk fills up to a specified point in the vessel. When a dealer gives less, there are complaints. Moreover, weight measures are typically converted to volume units in a similar fashion: a kilo of rice would again fill up to a specific level in the vessel. The fact that different levels obtain for different materials is to be pointed out. For example, a kilo of rice or a kilo of *atta* (flour), would measure up to different levels. This could be done practically in class.

Exercises

A variety of acclimatization activities are required at this stage, mainly to ensure that the learners are comfortable using standard measures:

• Get learners to make estimates in standard units: typical daily milk intake of an infant; typical height of a hut; the length and breadth of a postcard; the weight of a matchbox etc.

• Give numbers in standard units and ask what they mean to the learners. Exercises could be constructed in the following way: "Usha used up 12 litres of water". She needed this for:

-drinking,
-bathing,
-washing her face,
-washing a bucketful of clothes.

Similarly, with expressions like:
"Lata sat 20 feet above the ground". Most likely she was:
-sitting on a stool
-repairing the thatched roof of her hut
-plucking mangoes from a tall tree.

You can construct many different exercises with such statements as "Ruhi bought 2 metres of cloth", "He added 5 grams of sugar to..." etc. The exercises should give a set of interesting options for the learner to choose from.

Detail from a photograph by Henri Cartier-Bresson

• Bring a variety of objects and ask learners to guess their length, weight, volume etc. These objects can be almost anything found in the environment: vegetables, pencils, eggs, feathers, stones, tumblers of water, etc.

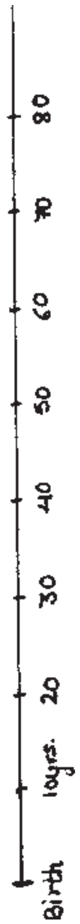
• Similarly, present two similar objects, and let the learners figure out which is longer, which is heavier, which has more quantity (volume) etc.

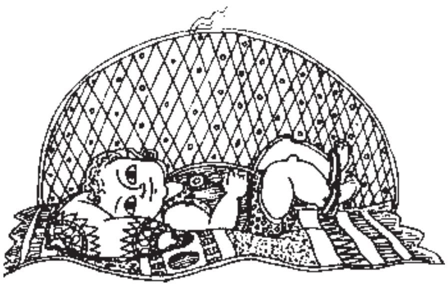
• Ask the learner to describe how each person in her family spends her time on a typical day, and write it out. From this, analyze how much leisure each person gets, how different people spend more/less time on the same thing etc. On the other hand, specify a time of day, and ask the learner to say what any person in the family is typically doing at that time. Correlate the two kinds of information.

• Draw a 'time line' for each person in her family. This should start with the time when the learner was born, and mark off important events on the line. Ask questions about durations between events.

• It is useful to give a set of puzzles and riddles involving measurement (see Part 2 of this book). All parts of India have a tradition of folk riddles of this kind, and including some of them in the primers would greatly enhance the learners' interest.

• Finally, the curriculum on measurement is completed with exercises that help the learners understand the formal relationship between traditional units and modem ones. These range from measuring a metre of cloth with one's hands to see how many cubits it makes, to making up a conversion table between these units.





My Time Line

What are the memorable events of my life. If we try to recall from the time we were born, each one of us can think of some events, both happy and sad, that have marked our lives. We could try to mark this on a time line.

Chapter 7

Basic Arithmetic and Applications

Addition and Subtraction

Situations

The first and most important step is the identification of addition/ subtraction situations from everyday life. Ask learners about specific situations where they need to add and subtract and ask what they do when faced with such situations. It is important to acknowledge that many learners already follow their own techniques to solve such problems in life and the numeracy class should indeed build on such knowledge.

Typical situations arise in shops where people buy several items and add up the cost of them all. For instance, a person buys a kilo of rice for Rs. 8/- and a kilo of sugar for Rs. 4/-. She needs to add and realise that she must pay Rs. 12/-. She has a ten rupee note and a five rupee note. Adding the amounts she realises she has Rs. 15/- with her. On giving these notes she must get back Rs. 3/- from the shopkeeper.

It is to be noted that both addition and subtraction arise in the situation outlined above. This is typical of most instances in everyday life. Hence an artificial separation of addition and subtraction exercises in numeracy classes only alienates learners from daily applications.

Other typical situations are:

• Wage addition: for the same person over a week, or for several people in the same day etc.

• Time addition: for instance, a dose of medicine is to be taken every 4 hours, and you have just taken it at 9:30 A.M. When is the next dosage due?

• Length addition: new clothes are being purchased for Diwali; it is usual to buy the same cloth material for all children in the family - estimate how many metres of cloth each child needs and add up to get the total requirement.

Such listing of situations is not merely a one-shot affair to be 'finished' before starting the 'real' addition 'sums', but something to be done everyday when addition exercises are done in class. This is important for the teacher also, because whenever there is any difficulty later with sums (like adding 23 and 37) they can be translated into situations thus identified, making it familiar to learners.

An artificial separation of addition and subtraction exercises in numeracy classes only alienates learners from daily applications.

Activities

Use of seeds and stones for addition/subtraction activities is absolutely a must. Get learners to make clusters of these and add. In such exercises, learners sit in groups. At first, give twenty seeds to each learner and ask each one to make 2 clusters, then count the number of seeds in each cluster. Record the 'addition facts' thus obtained on the blackboard (14 + 6, 17 + 3, 9 + 11 etc.). Repeat the exercise but telling the learners to ensure that no two persons within a group have the same 'break-up'. This can be repeated many times, varying the total number of seeds, as well as the number of clusters. A wide variety of addition facts generated thus are recorded in the centre's number diary.

Notice that the clustering activities involve both addition as well as subtraction. In fact, a list of corresponding subtraction facts can also be made.

Algorithms

• Encourage learners to use clustering to count. Give a large unspecified amount of seeds and ask them to count. While each seed can be counted out, it is easier to divide the lot into clusters, count each and record, and then add the recorded amounts.

• In shops, it is customary to add rather than subtract. For instance, when we give Rs. 20/- for bill of Rs.12/- it is usual for the shopkeeper to give a five rupee note, say '17', then a two rupee coin, say '19', then another coin, and announce the end of the transaction with '20'. Discuss this in class and ask the learners what they do.

• Using the nearest round number can greatly ease addition subtraction exercises. For instance, adding 18 to a number may be easier achieved by adding 20 and then subtracting 2. Similarly removing 18 is best achieved by subtracting 20 and adding 2. Ask learners to make up their own exercises for the use of such tricks.

• Get learners to make their own addition tables and use them to solve a given problem. Point out symmetries in these tables so that it is obvious to the learner that adding quantities in any order gives the same result.

When to do What:

Even when learners have understood how to add/subtract (in an algorithmic sense) they may often have difficulty in deciding when to do what. Given a word problem, it is important to determine whether the given situation demands for addition or subtraction. Typically, subtraction is recognized in situations where;

• A quantity is removed for some reason, and we need to determine what is left.
• Two (or more) entities are being compared, and we are trying to determine which is bigger and by how much.
• There is a deficit of some quantity and we must determine how much is additionally needed to overcome the deficit and make up the derived quantity.

Discussing a variety of such situations is essential, and without this learners will not achieve confidence in their numerical abilities. Learner's life experience should help them in making up such situations on their own.

Guesstimates

It is important to emphasize (at every stage) that most life situations call for not precise answers but good estimates. Therefore, when given any addition or subtraction exercise, even a learner who may make mistakes in answering it should be able to tell an interval in which the answer must lie. When told to add 52 and 28, any learner who knows that the answer cannot exceed 90 (as the given numbers are less than 60 and 30, respectively) is not going to write 710. At least, she will realise that 710 must be wrong and may say, "whatever the answer is, I don't know it, but it must be between 70 and 90."

Most life situations call for, not precise answers but good estimates.

This ability is important in life. For instance, while planning expenditures (for family/business) 6ne makes estimates for several items and finally makes a net estimate as an interval - 'we need at least Rs. 2000/- and hopefully not more than Rs. 2600/-.

Guessing can also provide a systematic method: when told to subtract 18 from 54, a learner may guess the answer to be '20', then add 18 and 20, see the result to be too small and hence revise the guess. A second attempt of 40' gives 58 which is too big, so she knows that the answer is between 20 and 40. A series of guesses like this is bound to lead to the answer. (This is called 'binary search' in Mathematics.)

Children of an African tribe play a game based on such a 'binary search' arrangement of stones. Sixteen stones are arranged in two rows of eight each. One person is sent away, and the others choose a stone. When the "out" person returns, he must determine which stone has been selected. He may ask four times, in which of the two rows the stone is located. After each reply, he may rearrange the stones within the two rows. He must be able to identify the chosen stone after the fourth reply.

The key to the solution lies in the procedure by which the stones are rearranged each time. After the first reply the questioner, rearranges eight stones, half the original number, so that they are not in different rows; after the second reply he interchanges four stones, half the previous number, and the next time he changes the position of two stones. The answer to the last question determines precisely which stone has been chosen.

(From: David Wells, *The Penguin Book of Curious and Interesting Mathematics)*

Carries and Credit

When adding two - digit numbers, the problem of carries is seen as 'difficult' in many numeracy classes. This is because top-down addition (with a number above and one below) is presented as a fait accompli to learners, with little motivation given, so that when a carry is placed on top of the 'left' digit, it seems like magic. The method needs to be motivated and explained so that this 'difficulty' is overcome.

In fact, we can start by giving an example of a common mistake.
"Ramaiyya wanted to add 22 and 4 and wrote

```
  22
+ 4
 62
```

The answer he got was clearly wrong. Why did he do so and how can we correct it?"

It is essential that addition of single digit numbers to two-digit numbers is done only with the use of fingers/quantities etc., for finding answers, and once the answer is already known, then (and only then) learners should be asked to write it top-down. Explaining this to be the convention. While doing so, it can be pointed out that:

• Adding say 22 and 4 amounts to adding only the right most digits and simply importing the other one.

```
  22
+   4
 62
```

• This demands that 4 be right-aligned

• Placing them aligned left

22
+ 4

simply does not work!

It is known to the learner that 25 and 6 give 31 (by counting). Writing this down in the new convention, we get

$$\begin{array}{r} 25 \\ +\ 6 \end{array}$$

Adding 5 and 6 give 11, and this results in the 'carry'.

This whole procedure is then explained as addition in two stages: 25 is 20 with 5 added, hence to add 6, we may keep 20 aside, add 5 and 6 to get 11, and then add it all to get 31.

Only when addition of single-digit numbers in thus clearly understood may we move to adding two double-digit numbers. Similar remarks hold for subtraction.
It is also useful to give exercises in sequences rather than in isolation, as they help get a mental picture of what is going on and can allow self-checking. Thus a learner who has trouble with 28 + 33 may find it easier to answer:

25	26	27	28	29	30
+ 33	+33	+33	+33	+33	+33

Exercises (addition-subtraction)

While we do use life-situations to motivate addition, subtraction etc., it is equally important to go beyond motivation and appreciate the use of acquired skills. When the learner has gained some competence and fluency with addition and subtraction involving numbers up to 1 00, she must test out this knowledge in life situations, and the class must consciously attempt to use the knowledge to help learners in a systematic manner.

A crucial use is in income - expenditure analysis, budgeting and account-keeping. Each of the learners should do this for her family, and the entire class may debate on the income - expenditure patterns of different families. It is to be emphasized that this should be done not as a 'mock' excercise but an earnest realistic one that may be of use to the learners.

A crucial use is in income-expenditure analysis, to be done not as a 'mock' excercise but an earnest realistic one that may be of use to the learners.

Another set of exercises involves age arithmetic. Each learner should systematically set down the age of every member of the family and calculate the year of birth of each person, marriage dates etc. A series of questions should be presented by teacher as well as learners involving age calculations. This is both fun and useful (see activity 'A Date with Numbers' in Part II, Section 4)

A greatly enjoyable group activity for the class is a menu making exercise - The entire class participates in deciding the menu for a dinner - they decide on the occasion, the guests, the menu, the required items, the necessary quantity of each etc. They also plan a schedule of how to prepare the items, work allocation etc. It is hard to separate out specific numeracy skills applied in such discussions, but they are definitely enhanced.

An Oral Riddle (Feast)

A couple in a house celebrated the ear-boring ceremony for their child. They had invited their relatives in their village for lunch. One hundred plantain leaves were laid out for the feast. As one of the items for lunch, they had arranged to fry one hundred pappads. They served the pappads differentially among men, women and children. Each man was served three, each woman two, and each child one half. A total of 100 persons ate in the feast. The hundred fried pappads have all been served. The persons and the pappads became even. Among those who ate, how many were men, how many were women, and how many were children?

(Answer 72 children, 20 women. 8 men)

Multiplication/Division

Multiplication is one of the routine activities everyone carries out in his/her life and yet it is often perceived to be 'difficult' in numeracy classes. This is because learners are rarely helped to see the connection between what they do routinely in life and what they learn in class. Thus again, the sequence of stages for 'teaching' multiplication/division is very similar to that adopted for addition/ subtraction:

• Discussion of life situations demanding these skills, - typically, shopping and wage calculations necessitate multiplication; estimation of large quantities (for instance, the yield of a mango or a tamarind tree, or the paddy yield of an acre), and so on.
• A whole series of activities with clustering of seeds, to get a feel for multiplication/division, and also to record 'multiplication facts' leading to learners forming their own tables. (For example, 12 x 3 = 36, 9 x 4 = 36, or 2 x 18 = 36)
• An emphasis on discovering algorithms already being used by learners and helping them discover 'standard' methods if they need them.
• Use of estimation and guessing in finding answers so that learners may also check their calculations.
The style of development is similar to that for addition and subtraction, so we mention only some specific points below.

Activities:

• Skip Counting

Skip counting is the basis of multiplication, and this can be introduced with a simple game: learners sit in a circle and start counting 1, 2, etc. If the number announced at the beginning of the game is 3, the counting goes 1, 2, bas, 4, 5, bas, 7, 8, bas, 10,......... Anyone who does it wrong (for instance the sixth person saying '6' or the seventh person, after hearing '5' and 'bas' saying '6') goes out of the circle and the next one takes up the count from these. This game is great fun because you cannot pre-decide what your number would be - as people leave in between, your number would change. It can be played many times with differing skip numbers.

While playing such games, it is a good idea to record the numbers on the board. This highlights the numbers which were skipped, namely the multiples of 3, and the teacher then points out that these are obtained by adding 3 successively.

Mixed Counting

A different version of this game is mixed counting. Here instead of 'bas', the day of the week has to be said in sequence. For instance, 1, 2, 3, 4, 'Monday', 6, 7, 8, 9, Tuesday', 11, 12, 13, 14, 'Wednesday' etc.

Clustering Activities

Clustering activities: making piles with equal number of seeds in each is the basic multiplication/ division exercise. There is no reason at all to force learners to make equal or unequal piles, calling it addition or multiplication. Learners keep doing the activity at their own leisure and pace. The special attention is only to point out to learners that given any number of seeds initially, we can always attempt to make it into equally sized piles, but that the attempt succeeds only for some numbers! Given that we are also discussing shop arithmetic all the time, an exercise of the form "make 5 piles with 8 seeds in each and see how many you use up" should easily connect with another one like "if a kilo of rice costs Rs 8/- what is the cost of 5 kilo of rice?"

• *Visual Patterns*

Visual patterns play a major role in understanding multiplication and division. Encourage learners to make patterns with seeds and also to draw them on their slates or notebooks. Nicely coloured patterns can be painted on thick paper or cardboard or hung up on the wall in the classroom. Kolams and rangoli use sophisticated patterns and figuring out the regularity in them is a good exercise for arithmetic as well. Simple patterns can be square, rectangular and triangular:

In each case, the learner should write row and column totals and also the 'grand' total. The fact that whether a rectangle is 'standing up' or 'lying down' the final total is the same, and connect it up with the fact that multiplying numbers in any order yields the same result.

It is also useful to see which are the numbers which lend themselves to a square, rectangular or triangular pattern. For instance, 6 can be set as a rectangle and a triangle, but not a square.

Use of sticks in multiplication can help greatly. To multiply 7 by 4 take seven sticks, arrange them horizontally, lay four sticks on them vertically, and count the points of contact.

It should be noted, and pointed out to learners as well, that such 'teaching aids' (like sticks etc.) are useful only to gain familiarity in classroom situations. In everyday life, repeated addition is the best algorithm available and a mental ability to do this needs to be cultivated.

In class, learners (working in groups) should be encouraged to 'derive' their own multiplication tables. These need not be memorised, but displayed prominently. It should be ensured that, given a problem, a learner can consult the tables and apply it to solve the problem on hand. If necessary, they should quickly derive the part of the table required, using repeated addition - for instance, when faced with a situation where 8 x 7 is required, the learner should compute

8,	16,	24,	32,	40,	48,	56
1	2	3	4	5	6	7

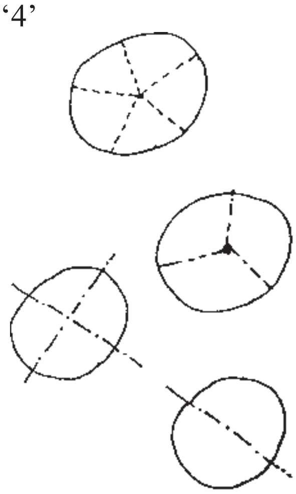
and give the answer.

An important point about division is to learn when to round up and when to round down.

Division

Use of tables for division is absolutely essential. Learners should be taught to consult tables for this: when 58 is to be divided by 9, the table for 9 is consulted. Though 58 is absent, the nearest number less than 58 in the table is 54, which gives 6 as the dividend. This should be immediately supported by an exercise to divide 58 seeds into piles each having 9 seeds and note that after 6 piles, 4 seeds are left over.

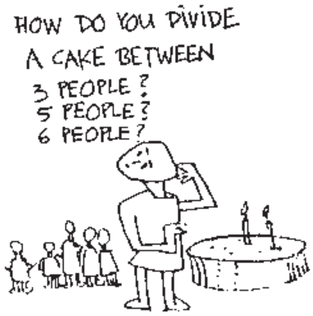
Another important point about division is to learn when to round up and when to round down. When we find that a boat can take 4 persons, and we are a group of 23, we do not say that we need 5 trips with 3 left behind, but round up and stress the need for 6 trips. On the other hand, when we want to know how many Rs. 5/-currency notes will get used up on giving away Rs. 22/- we ignore the 'left over' and say '4'



Consult the Learner

Like in the examples above (for Founding up/down), applying common sense is the most essential ingredient in numeracy lessons. Unfortunately, this is often a victim to the zeal for vigour and 'formal learning'. Adult learners can often supply much needed common sense and hence should be encouraged to freely criticise and remark on the meaning and relevance of exercises.

Learners' occupation and industry should provide the context for many exercises. For those who make woven mats, for weavers, for fishermen, for tillers, basic understanding will suggest many an exercise from their own routine, and that is the best curriculum they can possible get.



Making Parts

In primary schools, fractions constitute the part of mathematics syllabus that is perceived to be 'the hardest' for children. Often this extends to the TLC Volunteer Teacher as well, and her lack of confidence percolates to non-treatment or a cursory treatment of fractions in numeracy classes as well. This is indeed a pity, because adults understand basic fractions easily and use them in daily life extensively.

The first and most important step is to identify the vocabulary for fraction in the learners' mother tongue. Most languages have words for a quarter, half, and three-quarter. In Tamil, expressions like "three-quarter of a quarter" are in common parlance. A popular verse in old Tamil asks, "if you can get four-and-one-eighths banana for quarter-and-one-eighths paise, then how many can you get for a paisa?" (Note that one-eighths is standard usage in Tamil.) Similar idioms may be found in many Indian languages.

It is vital that extensive verbal discussion takes place in class about words like quarter, half, half of half, quarter of half without writing. Usually learners have good fun in such discussions and that is important. Written mathematical notation lacks the elegance of verse in this context and is often irrelevant to the adult neo-literate.

As usual, the fact that learners extensively use fractions by making parts in a variety of life situations needs emphasis. For instance, at harvest time, a fraction of the produce is set aside for the landlord, a fraction for God, a fraction for seeds, another for domestic use, and then the remainder set aside for commercial purposes.

Algorithms used by learners (or by others but familiar to learners) should be discussed in class. The most popular methods for dividing up grain are:

• **Piling**: Make piles of grain and adjust to make them equal : after making piles, recombine if required. For instance to find 2/5 th, make 5 piles first and then combine two of them.

• **Doling:** Make space for as many piles as required, mark the places and start doling, maybe one measure into each. At the end the result is approximately equal piles (but for the remainder).

• **Visual Estimates:** People extensively use visual judgment to decide whether a quantity is one-half of another. This could be supported by physical feel as well. Such physical estimation is as important as calculation and should be encouraged. The best way is to make such estimates, check them by calculation and develop confidence in one's estimating abilities.

It should be noted that the process of division and making fractional parts are one and the same things and thus understood traditionally in folk knowledge. In this, respect, a numeracy syllabus which teaches fractions and division separately is artificial. The important thing is to ensure that learners possess the ability to perform the necessary calculations that arise from life situations, so that learners should be able to use whatever algorithms they are comfortable with.

In understanding fractions, making parts of both discrete and continuous entities need stressing on. The former is as in piles of seeds - when we begin with, say 24 seeds and find out how many seeds are there in 2/3rd portion of the pile. To see an example of the latter, take a sheet of paper, fold it in half, tear off, again fold one of the pieces in half, and we have a quarter of what we began with.

Paper folding is an art in itself and simple fractions can be learnt well from origami exercises. Numeracy classes can include simple origami lessons; they are fun and educative.

Arithmetic with fractions is often difficult and for purposes of basic numeracy it suffices to stick to fractions of the type: 1/2, 1/4, 1/8, 1/3, 2/3, 3/4, 1/5, 2/5, 3/5, 4/5. Use clusters of 24, 48, 60 seeds as these numbers have several factors.

Applications

Once learners have obtained fluency with the four basic arithmetical operations, the class must take up a serious study of the professions of each of the learners present, and consciously look for numerical applications therein. Exercises common to everyone:

Shop Arithmetic: The class can enact a shop: they decide on what items are sold in the shop at what unit price and how much stock is available in the shop. With one person playing the shopkeeper's role, the rest come with shopping lists. The seller prepares bills which are checked by the buyers. Learners should start asking for a bill in shops thereafter and check the bills from then on.

The numeracy curriculum should explicitly include a shop transaction syllabus, because one important aspect of modernization is in involving every person, literate or not, in purchase of goods produced by remote agencies. Learners should be empowered to:

• Ask for bills, demand publicised price lists, and check that the quantities and prices charged in the bill tally with those announced in the shop.
• Check, in the case of manufactured goods, the suggested maximum retail price on the packet with what they pay.
• Check, in the case of medicines and edible goods, if any expiry date is mentioned on the packet. If yes, they should ensure that the purchase date is earlier.

Domestic/Business Accounts: All learners attempt to maintain a systematic account of money coming in to their family as well as money going out and attempt to 'balance the books'. Clear statements of account prepared for 3 successive months can give a very clear idea of how money is being spent. In fact, before starting this, the learner should be asked for an estimated statement of her family's income-expenditure pattern and this should be compared against actuals. Many neo-literate centres have seen an enthusiastic response from learners for this exercise and usually learners claim that it helps their understanding considerably.

Festival Budgeting: It is an important aspect of such family finance exercises. On a special occasion the family tends to spend large sums and neo-literate women often are found to lack a clear understanding of how the money is spent. Conscious budgeting with the help of the class would greatly help such learners, and often unforeseen economisation may result due to discussion.

Many of the learners are involved in small businesses, and simple accounting procedures may help them manage these activities better.

Optimisation Exercises: Group discussions are both necessary and most helpful in these exercises which form the best applications of numeracy skills. The idea is to start with resource constraints and arrive at best possible utilization of available resources. Concretely, the following examples may suggest the pattern of exercises.

As described above, the class enacts as a shop. The difference now is that the buyers are told a fixed amount (common to all) and plan their purchases so that they spend the amount in the manner they consider best. The buyers are encouraged to spend all the 'money'. After their 'purchase', a discussion takes place both on how well people had planned and on the manner of expenditure.

The class also plans-for the shop. Since the items sold in the shop are decided in the class itself, they consider each item in turn and discuss the profitability of selling that item. They decide how often the stock of each item is to be replenished. This necessitates consideration of which goods are perishable, which need to be fresh for sale, etc. Many neo-literates are involved in the sale of small merchandise like vegetables, and deciding on a proper mix of what vegetables they would buy from the (whole sale) market is an area where numeracy skills can be of help. More than the arithmetical operations, the very act of putting the options down on paper and considering alternatives 'formally' helps to clarify issues and suggests appropriate decisions. The class may enact running of small kiosks, selling household items, vegetables, fruits, milk and dairy products, and other edibles.

Menu Making Under Fixed Constraints: The class returns to menu making exercises but this time with fixed budgetary constraints. Learners are told how much money is available, how many people are eating and what the occasion for the meal happens to be. The class now has to consider various menu options, work out the estimated cost of each and decide on which is the 'optimal' menu within the budget. Along with the menu, the class should also work out the recipes for making the items, as these also call for applications of numeracy skills.

Chapter 8

A Panchayat Activity: The Metric Mela

Even as you join the queue, you ask why 'metric' mela, but receive no clear response. The volunteers, young girls themselves, giggle a bit and are mysterious.

Even as you approach the mela, you can hear the songs on the loudspeaker. You wish somebody would reduce the volume, but it is undeniable that the din caused does add to the festive atmosphere. Indeed it seems to go with the festoons, streamers and the general riot of colour, with noisy children running about adding to the mela mood.

A couple of volunteers come forward to welcome you, assuring that you are about to have a "totally new" experience, and that this mela is entirely run by their students, the neo-literates of the village. You have heard this before, when they went door to door yesterday inviting everyone in the village to come, and even offered attractive prizes! What was it, yes, they said - so many prizes will be given away, it will be a big surprise if you don't get any!

Even as you join the queue, you ask why 'metric' mela, but receive no clear response. The volunteers, young girls themselves, giggle a bit and are mysterious - you'll find out soon.

You reach the 'registration desk' and Muniyamma is presiding over it. It is interesting to see her there, she sells you milk everyday. She hands you a red card, about the size of an A4 sheet, and asks for your name and address, (as if she didn't know it!). While you fill up, she enters the name and address in her own white register, decides a number for you - 420, enters it in big letters on the top right hand comer of your card, and you are pushed along "Next!"

You see that the meal is arranged in a series of stalls, and the queue is inching its way through them. There are more than 20 stalls, you estimate. Even as you move along, you are intrigued by your red card, and you take a look at it. It has a big table with each row having a description and some blank entries. Passing over routine items like height, weight etc. you are intrigued by entries like 'weight of cabbage',' length of lauki' etc. Wait a minute, what is that one - 'length of nose'?! They are not going to measure the length of your nose, are they!?

The Card

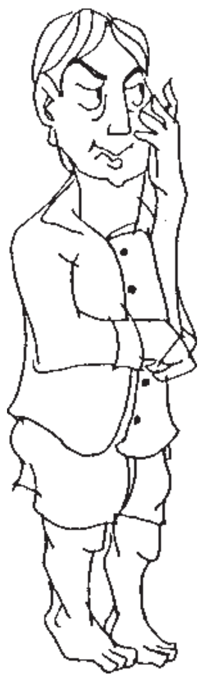
'Weight of cabbage', 'length of lauki', 'length of nose'? They are not going to measure the length of your nose, are they ?

Name : Male/Female Card No..
Address : Adult/Child
Height : Cms Weight: **Kgs**
'My' weight : 'My' height:
Length of 'lauki' : Length of Nose:
Length of Chalk : Little finger:
Distance to Meenakshi's hut: Weight of a Cabbage:
Weight of an Egg : Weight of a Dal packet:
Weight of a feather: Which has more water?:
Volume of water in a bucket: Volume of water in a bottle:
No. of items cored in memoy test: No. of stones:
No. of leaves: Time for counting:
Guess the item- 1 : 2: 3 :

The first stall you go to has a person with a measuring tape, who measures your height, enters it in your card and makes an entry in her own register as well, along with your card number. In the next one, as expected, someone measures the length of your nose! They have made a 'nose mask[1] which is placed on your nose and die length is measured on that one, so that the measurement is reasonably accurate. You want to know where you can buy such calibrated nose masks, and you are told that they will make it for you for a nominal fee!

This is the pattern as you move" from stall to stall - something measured, or you are asked to guess the length, weight or volume of a given object. When a chicken feather is given, you have hardly any idea of what its weight might be, and try to peep into the register, to see what estimates other people have given. Of course, you are caught at it, and pushed away among roaring laughter. After much hesitation, you think hard, then blurt out an estimate, and the knowing smile you get on hearing that tells you that your estimate is indeed wildly off the mark.

The counting games are also interesting - in some you are given a time limit (20 seconds) and asked to guess the number of leaves/ stones/..... within that time, whereas in others, they see how long you take to count out a certain quantity. There are also memory tests and conversion games. In the latter, you are presented with pictures with data in some units and then asked a question in different units.



The neo-literate who runs the stall, talks about how many got closer to the answer, how many gave wild guesses...

There are also some stalls where you can buy sliced fruit, peanuts, chikki etc. You are surprised to find that entries are being made in these stalls as well. Near the main area but outside the stalls region, there is an exhibition, mainly consisting of displays of items prepared by neo-literates. You are greatly impressed by the variety of talent on display.

While you now have a pretty good idea of what the mela is about, and why it is a metric mela, the whole thing falls into place only in the evening, when there is festive cultural programme followed by the much - awaited prize-giving ceremony. This is a virtual riot as there are prizes in most interesting variety. There is one for the person with the longest nose, and one for the person with the shortest nose. The person who got the closest estimate for the weight of cabbage gets the cabbage itself as a prize, and similarly the one who got the length of the drumstick right gets the drumstick. Prizes for the tallest, shortest, heaviest, lightest,........... Indeed there are prizes for almost everyone, and you get the prize for......getting the weight of the chicken feather correctly!

The prizes are given away by the neo-literate who ran the stall, and this is in itself a novel experience for her and for the village. She also talks about how many got close to the answer, how many gave wild guesses (with some examples, causing much mirth). The ones who sold goods present an account of the sales and the profit they made.

Interestingly, in all the stalls, though people told their answer in whatever units they pleased, they have been converted to metric units before recording. Thus the term 'metric mela!'

This vivid description is from the written notes of one of us (R.R.) who actually participated in this Metric Mela. Many such Melas have been held in some of the TLC districts of Tamil Nadu. The entire planning and organisation was done by the Volunteer Teachers and the neo-literates themselves, with the help of literacy activists and resource persons. Not only does such an activity boost the morale of all those involved in the TLC, but equally significantly, it also motivates school children to 'practice' their school knowledge of mathematics in a more meaningful and enjoyable manner. They are the 'resource persons' for a change, who have to supervise all unit conversions, all measurements, computations, etc, and this gives them unprecedented joy and confidence. In turn it reassures their neo-literate parents and reinforces their belief in the importance of schooling and literacy for the whole community.

The spirit in such Melas was truly infectious we came back feeling convinced about the need to launch a large-scale Panchayat 'Numeracy' Campaign to rejuvenate the earlier TLCs. We hope this book would enthuse some people to move in such a direction, taking 'numeracy' to be the new focal theme.

The spirit in such Melas was truly infectious; we came back feeling convinced about the need to launch a large-scale Panchayat 'Numeracy' Campaign to rejuvenate the earlier TLCs. Moreover, such a Panchayat activity assumes greater significance in the post-literacy context, of looking for a more decentralised model for the literacy campaign focused at the Panchayat level. We hope this book would enthuse some people to move in such a direction, taking 'numeracy' to be the new focal theme.

A Feel for Large Numbers: Towards a Panchayat Plan

Large Estimates

There is an important side to preparing such estimates involving large numbers : they help in planning panchayat action for local Government as well as in initiating local enterprises. Today many gram panchayats in India are peopled by neo-literates and estimating the requirements of the entire village calls for such exercises by them. In a situation where most panchayats spend whatever finances available to them in *an adhoc* manner, empowering panchayat members with an ability to estimate requirements, plan expenditure and budget for it, can make for major changes.

In terms of enterprise, a calculation of how much milk could be used per day in the village, the number of cows required for supplying such a quantity of milk and the number of cows present, can suggest the feasibility of getting additionally required cows and start a business of supplying milk. Admittedly there is more to initiating enterprises, but this is a necessary first step.

Moreover, plans for participatory resource management necessarily involve the village community through the panchayat members. For instance, as part of many PL programmes in different blocks of the country, "watershed management" has been taken up as a major programme. For the community to estimate its own water needs, to accurately map all its possible water resources, and to chart out a plan for future water harvesting would require some level of confidence in its own numeracy skills.

Large Estimates and Fermi Questions

While the basic numeracy curriculum works with only small numbers, typically less than 100, learners do need and have a basic understanding of large numbers as well. However, adding two 5-digit numbers like 53642 and 24864 is an entirely irrelevant exercise. What is important is that learners understand that

• We count in hundred, thousands, lakhs and crores,
• These arise systematically, and arithmetic with such numbers is in principle no different from what has been leant, but only more tedious.
• Place-value notation helps in this.

The words thousand, lakh etc,"are usually well-known to learners, though the associated quantities are difficult to imagine.

Number of Leaves on a Tree

Some simple exercises help in visualising how numbers grow. Ask the class how many leaves a particular neem tree (standing nearby) has. Usually estimates vary wildly. emphasize that in such situations the exact number is irrelevant, but we need to get the answer correct to the nearest hundred or thousand as the case may be. (This is because, if a tree has more than 20,000 leaves, getting a thousand wrong would still mean less than 5% error.) Now cut out a small branch of the tree, count the leaves in it systematically, and round up the number to a convenient figure. Suppose this is 50. Now estimate how many such small branches the tree has. This again typically necessitates viewing the tree as having several big branches, each with several small branches. For instance, consider the big branch from which we plucked the small branch. Perhaps this has branches in clusters on either side, each cluster having roughly 8 branches. If there are 5 clusters, on each side there would be 40 branches, thus giving 80 small branches in that big branch. If there are 10 such big branches in the tree, we get 800 small branches in the whole tree, giving us a good estimate of 40, 000 leaves in the tree.

This kind of estimation is fairly common in rural India where estimates of likely yield of a tamarind tree or mango tree are involved. Leasing of trees is a tradition and the base amount is fixed in this way.

While an exercise like this helps learners see how numbers grow, the number of zeroes may confuse them, so it is important to write '40 thousand' rather than '40000' and so on. Place value notation, distinguishing 5,50,500,5000 etc. can be emphasised at this stage.

Population Estimates

A reinforcement exercise in this regard is the study of the village population. Learners are invited to map out their village in terms of neighborhoods, then each into street, then each streets into households. Now starting with a typical number of persons in the household, they arrive at an estimate of the number of persons living in the village. Based on this, they estimate various figures like-

• Number of children in the village below 10 years,
• Number of women in the village in the age group 15-45,
• Number of cows and buffaloes in the village and so on. If possible, data can be gathered of the corresponding actual figure so that these estimates can be checked.

Such estimation naturally leads to so-called **Fermi questions** (after the great physicist Enrico Fermi who not only loved to pose such questions but whose estimates always showed uncanny accuracy). For instance, how many cups of tea were drunk in Delhi today ? Or, how many people watched television for more than an hour today in India? Coming up with similar question relevant to the neo-literates is easy, and answering them is a good group activity which is not only enjoyable but gives the group a better understanding of the world. A neo-literate woman calculating the number of chapattis she had made in all her life till then, claimed that the exercise gave her a new outlook on life!

Note :

Training of VolunteerTeachers and Resource Persons

Traditionally, training in TL/PL campaigns have tended to focus more on literacy skills rather than numeracy. Reasons for this have already been discussed in chapter 2. This handbook presents a new perspective for numeracy in TL/PL campaigns and this will also involve a change in the way training is conducted. Here are some important points to be kept in mind:

• Resource persons must undertake a preliminary study of:
(a) the numeracy vocabulary in vogue in the region,
(b) local folk algorithms used for arithmetic and measurement,
(c) typical numeracy requirements in professions practiced by neo-literates.

The depth and seriousness of such a study will be limited by the time and quality of resource persons available, but yet this needs to be done, not only for the result obtained, but also to sensitise them in the process. In a sense, the study is itself part of the training for resource persons in numeracy.

• An elaborate training schedule covering arithmetical operations, fractions and measurement may be welcome to refresh the volunteer teachers' own knowledge and understanding of the subject. For many VTs, this would indeed be needed.

• The activities suggested must actually be carried out during training. Only then will their use become clear.

• The training must concentrate on orienting the teachers' attitudes so that they actively look for what is already known to the learner and build on it, and so that they can assess the learner's requirements.

• The training must attempt to equip the teacher with the ability to make up her own exercises on the basis of those given in this book rather than exclusively rely only on the book exercises.

> This handbook presents a new perspective for numeracy in TL/PL campaigns and this will also involve a change in the way training is conducted.

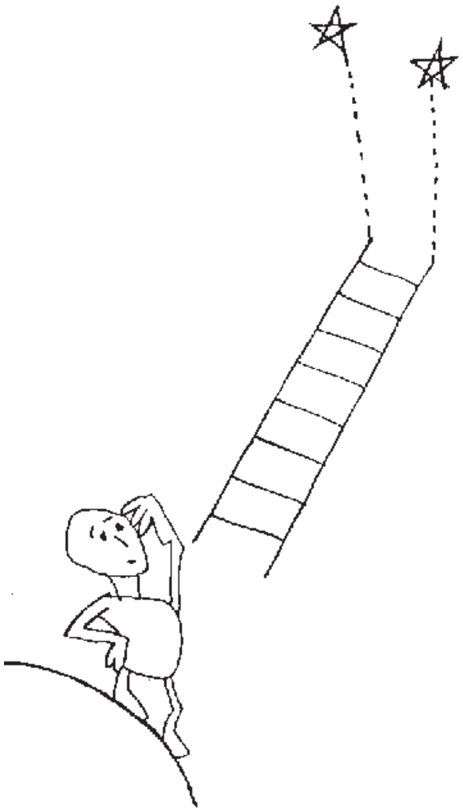
A 2-Day Schedule For Training Of Volunteer Teachers.

Day 1

09:30 am Introductions
10:30 am Presentation-Numeracy in TL/PL Campaigns, Oral and Written Arithmetic
11:00 am Tea Break
11:15 am Group Discussion: "What learners know, what they don't and what they need to know"
12:15 pm Consolidation of Group Discussions
01:00 pm Lunch Break
02:00 pm Group Work: Basic Arithmetic and Applications
05:00 pm Group Games - Applications
06.00 pm The Metric Mela, An Actual Pilot Exercise

Day 2

09:00 am Local Songs, Riddles and Stories for Numeracy
10:00 am Presentation: Folk Knowledge in Measurement
11:00 am Tea Break
11:30 pm Group Work: Measurement
01:00 pm Lunch Break
02:00 pm Group Work: Designing New Exercises and Activities
04-05 pm Review and Feedback.



Part II

Riddles, Puzzles and Stories

Section 1: Oral Riddles from Tamil Nadu

1.1 Broken Eggs:

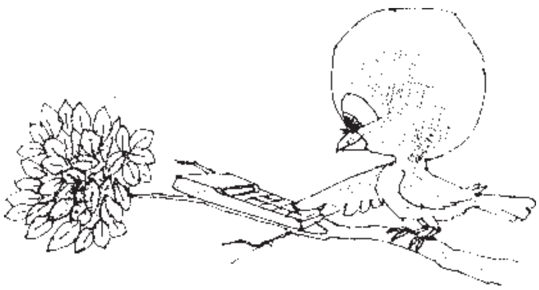
An egg trader was moving along a road selling eggs. An idler who didn't have much work to do started to get the egg trader into a wordy duel. This grew into a fight and he pulled the basket with eggs and dashed it on the floor. The eggs broke. The trader requested the Panchayat (five member committee) meeting to settle the dispute. This Panchayat asked the trader how many eggs were broken? He gave the following response:

If counted in pairs, one will remain;
If counted in threes, two will remain;
If counted in fours, three will remain;
If counted in fives, four will remain;
If counted in sevens, nothing will remain;

(119 eggs)

1.2 A Flock of Sparrows:

A sparrow was sitting on a branch of a tree. At that time a flock of sparrows was flying above the tree. The sparrow that was sitting called out for the flock of sparrows: Oh! One hundred sparrows! Oh! One hundred sparrows!! Where are you going? come and sit on this tree and take rest and then continue your journey. Hearing this, one of the sparrows from the flock said, "We are not hundred. We, a similar flock like us, one half of that, one half of that and you together will make one hundred". If (hat is so, how many sparrows were there in the flock that was flying?



It would be interesting to see how non-literate people solve this riddle orally. We could try to understand the strategies they use. But here we give a straight forward solution using an algebraic equation. If n is the number of sparrows in the flock, then :

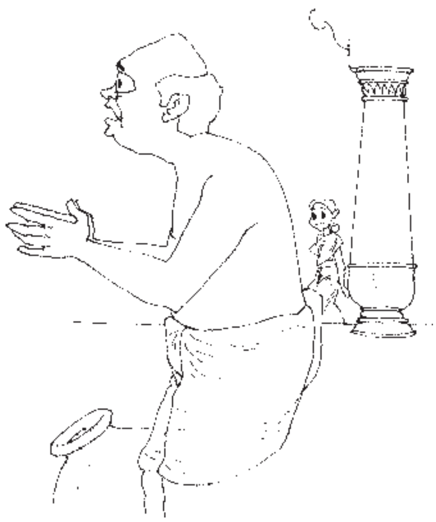
$n + n + n/2 + n/4 + 1 = 100$ or
$2n + 3n/4 = 99$
$11n = 396$

so that $n = 36$

1.3 Oil Merchant:

In a village, there lived an oil merchant, who used to crush oil seeds, extract oil and sell it. After several days of hard work, he got ready to take the extracted oil to the market. On his way to the market he came across a Vinayaka temple. He entered the temple and prayed "Today, if my business goes well, on my way hack, 1 will light the lamps in the temple using one measure of oil."

He moved on. As he crossed some more distance, he saw a temple of the Goddess. He went in and again prayed the same way he prayed to Vinayaka. He continued his journey and reached the Ayyanar temple at the village limits. He again prayed the same way.



He went to the market. He had good business that day. In the evening as he was returning, he took sufficient quantity of oil in a pot to fulfill his vow in the three temples. He reached the Ayyanar temple. He saw a small tank of water near the temple, He kept the pot on the tank-bund and he got into the tank in order to wash his feet, hands and face. At that time a crow came and sat on the pot and tilted it. The oil flowed out on the ground. The merchant came running and straightened the pot. A small quantity of oil remained in the pot. A good quantity of oil was spilt on the ground.

With the quantity of oil left in the pot, the merchant went to the Ayyanar lemple. He expressed his feelings of not being able to fulfill his promise. Ayyanar sympathised with him and blessed him saying "May the quantity of oil in the pot be doubled." And so it happened. The oil trader fulfilled his vow of lighting lamps in the temple using one measure of oil.

He covered some distance. He reached the temple of the Goddess with the pot containing little quantity of oil. Here again he narrated the happenings and expressed that he felt sorry that he was unable to do what he vowed. The Goddess blessed him by doubling the quantity of oil in the pot (he merchant took out one measure of oil and lighted the lamps and fulfilled his vow. Little quantity of oil was left in the pot.

The merchant walked further and reached the Vinayaka temple. With all devotion he narrated all that happened and expressed with sadness his inability to fulfill his vow. Vinayaka also blessed him with double the quantity of oil. The merchant took out one measure and lighted the lamps. The pot was empty now.

What was the quantity of oil in the pot when the merchant straightened the pot after driving the crow away?

[seven eighth measure (7/8)]

1.4 Milk Vending:

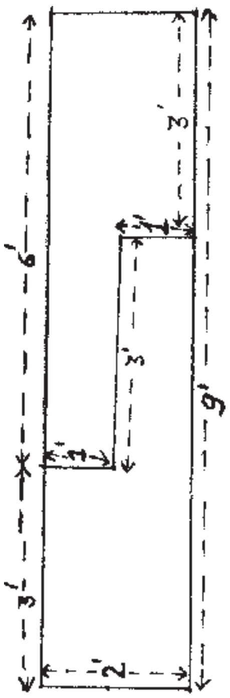
A milk vendor had several customers to whom he was selling milk everyday. He generally brought milk in big metal pots and measured with traditional standard volume measures and supplied any quantity between one measure to eight measures. One day he forgot to bring in his standard volume measure. The customers also didn't have any clean volume measure for him to use. The capacity of the two metal pots he had been in terms of three measures and five measures. He used these two metal pots and measured milk from one to eight measures. How did he do it?

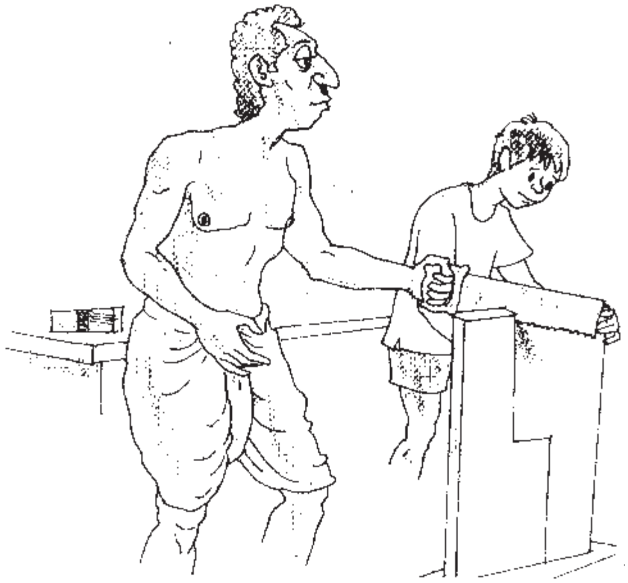
1 measure	= (2 x3)-5=l
2 measures	= 5 - 3 =2
3 measures	= 3 =3
4 measures	= 2 (5-3) =4
5 measures	= 5 =5
6 measures	= 2 x 3 = 6
7 measures	= 5+ (5-3) = 7
8 measures	= 5 + 3 = 8

1.5 Broken Weight Measure:

A 40 palams (one viss) weight measure dropped down. It broke into four pieces. With these four pieces it was possible to weigh things from one palam to 40 palams. How much was the weight of each of the broken pieces?

(1, 3, 9, 27 palams)





1.6 Changing the Dimensions of a Plank:

A plank is of 6 feet length and 3 feet width. It has to be made into a plank of 9 feet length and 2 feet width. It is to be cut along a single continuous lineonly. There should be only one joint. How to cut and join?

(The plank can be cut along the line in the centre as shown in the figure and the piece on the right is to be joined with the piece on the left so as to make it a rectangle. If can be tried out with pieces of paper.)

Section 2. A Feel for Giant Numbers

2.1 Legend about the Chess-Board

Chess is one of the world's most ancient games. [t has been in existence for centuries so it is no wonder that it has given rise to many legends whose truthfulness cannot be checked because of the remoteness of the events. One of these legends I want to relate. You do not need to be able to play chess to understand it, it is sufficient for you to know that it involves a board divided into 64 cells (black and white alternately).

The game of chess was invented in India. When the Indian king Sheram got to know about it he was amazed at its ingeniousness and the infinite variety of positions it afforded. Having learned that the play was invented by one of his subjects, the king summoned him in order to reward him personally for such a stroke of brilliant insight.

The inventor, named Seta, came before the sovereign's throne. He was a simply dressed scribe who earned his living giving lessons to pupils.

"I want to reward you properly, Seta, for the beautiful game you invented," the king said.

The sage bowed.

“I’m rich enough to fulfill any of your desires,” the king went on to say. “Name a reward that would satisfy you and you’ll get it”.

“Don’t be shy! What’s your desire? I’ll spare nothing to meet your wish!”

“Great is your kindness, oh sovereign. Give me some time to sleep on it. Tomorrow, upon consideration, I’ll name you my wish!”

When the next day Seta came to the throne he amazed the king by the unprecedented modesty of his desire.

Seta said: “Sovereign, please order that one grain of wheat be given to me for the first cell of the chess-board.”

“A simple wheat grain?” the king was shocked.

“Yes, sovereign. For the second cell let there be two grains, for the third four, for the fourth eight, for the fifth 16, for the sixth 32....

“Enough!” the king was exasperated. “You’ll! get your grains for all the 64 cells of the board according to your wish: for each twice as much as for the previous one. But let me tell you that your wish is unworthy of my generosity. By asking for such a miserable reward you show disrespect for my favour. Truly, as teacher you might give a better example of gratitude for the kindness of your king. Go away! My servants will bring you the bag of wheat”.

Seta smiled, left the hall and began to wait at the palace gates.

At dinner the king remembered about the inventor of chess and asked if the foolish Seta has collected his miserable reward.

The answer was: “Sovereign, your order is being fulfilled. The court mathematicians are computing the number of grains required.”

The king frowned - he wasn’t used to having his orders fulfilled so slowly.

At night, before going to bed the king Sheram again inquired how long before had Seta left the palace with his bag of wheat.

“Sovereign, your mathematicians are working hard and hope to finish their calculations before dawn.”

“Why so long?” the king was furious. “Tomorrow, before I wake up everything, down to the last grain, must be given to Seta. I never give my order twice!”

First thing in the morning the king was told that the chief mathematician humbly asked to make an important report.

The king ordered him in.

Sheram said: "Before you bring out your business I'd like to know if Seta has at last received the miserable reward that he asked for."

The old man responded: "It's exactly because of this that 1 dared **to** bother you at such an early hour. We've painstakingly worked out the number of grains that Seta wants to have. The number is so enormous...."

"No matter how enormous it is", the king interrupted him arrogantly, "my granaries won't be depleted! The reward is promised and must be given out....."

"It's beyond your power, oh sovereign, to fulfill his wish. There is not sufficient grain in all your barns to give Seta what he wants. And there is not enough in all the barns throughout the kingdom. You would not find that many grains in the entire space of the earth. And if you wish to give out the promised reward by all means, then order all the kingdoms on earth to be turned into arable fields, order all the seas and oceans dried up, and order the ice and snowy wastes that cover the far northern lands melted. Should all the land be sown with wheat and should the entire yield of these fields be given to Seta, then he'd receive his reward."

The king attended to the words of the elder with amazement. "What is this prodigious number?"

"18, 446, 744, 073, 709, 551, 615, oh sovereign!"

Such was the legend. There is now no way of knowing if it's true, but that the reward is expressed by this number you could verify by some patient calculations. Starting with unity you'll have to add up number 1, 2, 4, 8, etc. The result of the 63rd doubling will be what the inventor should receive for the 64th cell of the board.

If you want to imagine the enormousness of this numerical giant just estimate the size of a barn that would be required to house this amount of grain. It's known that a cubic metre of wheat contains about 15,000,000 grains. Consequently, the reward of the inventor of chess would occupy about 12,000,000,000,000 cubic metres, or I2,000 cubic kilometres. If the barn were 4 metres high and 10 metres wide its length would be 300,000,000 kilometres, twice the distance to the Sun!

The Indian king could never grant such a reward. Had he been good at maths, he could have freed himself of the debt. He should have suggested to Seta to count off the grains he wanted himself.

In fact, if Seta kept on counting day in day out he would have counted only 86,400 grains in the first 24 hours. A million would have required no less than 10 days of continual reckoning, and thus to process 1 cubic metre of wheat would have required about half a year. In ten year's time he would have handled about 20 cubic metres. You see that even if Seta had devoted a lifetime to his counting, he would still have only obtained a miserable fraction of the reward he desired.

2.2 Crazy Multiplication: A feel for 'Ecological Balance':

A ripe poppy head is full of tiny seeds, each of which can give rise to a new plant. How many poppy plants shall we have, if all the seeds germinate? To begin with we should know how many seeds there are in a head. A boring business, but if you summon up all your patience you'll find that one head contains about 3,000 seeds.

What follows from this? If there is enough space around our poppy plant with adequate soil, each seed will produce a shoot with the result that the following summer 3,000 poppies will grow. A whole poppy field from just one head.

Let's see what will happen next. Each of the 3,000 plants will produce no less than one head (more often several heads), with 3,000 seeds each. Having germinated, the seeds of each head will give 3,000 new plants, and hence during the following year we are going to have

3,000 x 3,000 = 9,000,000 plants.

Calculation gives that in the third year the offspring of our initial head will already reach

9,000,000 x 3,000 = 27,000,000,000

In the fourth year there will be

27,000,000,000 x 3,000 = 81 ,000,000,000,000offspring's.
In the fifth year our poppies will engulf the earth, because they'll reach the number

81,000,000,000,000 x 3,000 = 243,000,000,000,000,000.

But the surface area of all the land, i.e. all the continents and islands of the earth, amount to 135,000,000 square kilometres, or 135,000,000,000,000 square metres - about 2,000 times less than die number of the poppy plants grown.

You see thus that if all the poppy-seeds from one head germinated, the offspring of one plant could engulf the earth in five years so that there were about 2,000 plants for each square metre of land. Such a numerical giant lives in a tiny poppy seed!

A similar calculation made for a plant other than the poppy, one which yields less seeds, would lead to the same result with the only distinction that its offspring would cover the lands of the earth in a longer period than five years. Take a dandelion, say, which gives about 100 seeds annually. Should all of them germinate, we should have;

Year	**Number of Plants**
1	1
2	100
3	10,000
4	1,000,000
5	100,000,000
6	10,000,000,000
7	1,000,000,000,000
8	100,000,000,000,000
9	10,000,000,000,000,000

This is 70 times more than the square metres of land available on the globe.

In consequence, the whole earth would be covered by dandelions in the ninth year with about 70 plants on each square metre.

Why then don't we observe in reality these tremendous multiplications? Because the overwhelming majority of seeds die without producing any new plants, they either fail to hit a suitable patch of soil and don't germinate at all, or having begun to germinate are suppressed by other plants, or are eaten by animals. If there were no massive destruction of seeds and shoots, any plant would engulf our planet in a short period.

This is true not only of plants but of animals, too. If it were not for death, the offspring of just one couple of any animal would sooner or later populate all the land available. Swarms of locust covering huge stretches of land may give some idea of what might happen on earth if death didn't hinder the multiplication of living things. In two decades or so the continents would be covered with impenetrable forests inhabited by uncountable animals struggling for their place under the sun. The oceans would be filled to the brim with fish so that any shipping would be impossible. And the air would not be transparent because of (he mists of birds and insects...

Before we leave the subject, we'll consider several real- life examples of uncannily prolific animals placed in favourable conditions.

a) At one time America was free of sparrows. The bird that is so common in Europe was deliberately brought to the United Slates to have it exterminate the destructive insects. The sparrow is known to eat in quantity voracious caterpillars and other garden and forest pests. The sparrows liked their new environment, since there were no birds of prey eating them, and so they began to multiply rapidly. The number of insects began to drop markedly and before long the sparrows, for want of animal food, switched to vegetable food and went about destroying crops. The Americans were even forced to initiate a sparrow control effort which appeared to be so expensive that a law was passed forbidding the import to America of any animals.

b) There were no rabbits in Australia when the continent was colonized by the Europeans. The rabbits were brought to Australia in the late 18th century and as there were no carnivores that might be their enemies; they began to multiply at a terrifically fast rate. Hordes of rabbits soon inundated Australia, inflicting enormous damage to agriculture. They became a plague of the country and their eradication required great expense and effort.

c) A third instructive story comes from Jamaica. The island was suffering from an abundance of poisonous snakes. To get rid of them it was decided to introduce the secretary-bird, an inveterate killer of poisonous snakes. The number of snakes soon dropped all right, but instead the island got to be infested with the rats that earlier were controlled by the snakes. The rats wrought dreadful havoc amongst the sugar cane fields and posed an urgent problem. It's known that an enemy of the rat is the Indian mongoose, and so it was decided to bring four pairs of these animals to die island and allow them to multiply freely. The mongooses adapted perfectly to their new land and in a shon period of time inhabited the island. In less than a decade they had almost wiped out the rats. But alas, having destroyed the rats, the mongooses began to consume whatever came their way and turned into omnivores. They started killing puppies, goat-kids, piglets, poultry. And when they had multiplied still further they set about devastating orchards, fields and plantations. So the inhabitants of the island were compelled to start combating their previous allies, but with limited success.

d) In India we have had similar experiences. The most well-known examples are parthenium or the "Congress grass" (also called "gajar ghaas") weed and the 'water hyacinth' plant which have proliferated so wildly that a major problem has cropped up. The 'gajar ghaas' is known to cause severe allergies, while the 'water hyacinth' has covered most ponds and rivers, causing them to dry up.

More of such local examples could be discussed in class.

(From: *'Fun with Maths and Physics',* Y.L. Perelman, Mir Publications, 1994)

2.3 Don’t Hit the Ceiling - With A Pile of Papers!

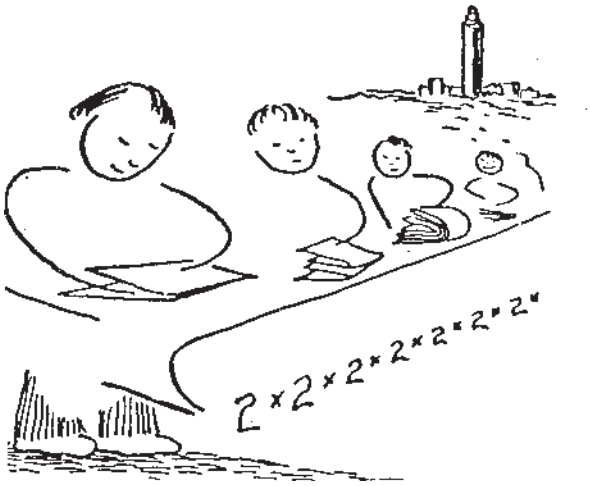
Let us try another one, and this time give it a little more thought: Suppose you had a paper say about three-thousandths (0.003) of an inch thick.

Now lay another similar paper on top of it; the two will of course be twice as thick as one,

0.003 x 2 = 0.006

or six-thousandths of an inch thick.

Now put two more papers on top of that making 4 in all,
which are 0.003 x 4 = 0.012,
or twelve-thousandths of an inch thick.



Continue this process,
each time doubling the number of papers,
thus:
the first time you had 1 paper,
the second time you had 2,
the third time, 4
the fourth time, 8,
the fifth time, 16,
and so on,
doubling the number each time,
as we said before.
Now continue this 32 times.
The question is:
HOW HIGH WILLTHE PILE OF PAPERS BE?
Do you think it will be 1 foot high?
Or will it be as high as
a normal room, from floor to ceiling?
Or as high as the Empire State Building?
Or what?
The correct answer is not necessarily
any of these
What do YOU think?
Let us again make out a table,
showing clearly what was done:

	Number of Papers	Thickness
1st time	1	.003 in.
2nd time	2	.006 in.
3rd time	4	.012 in.
4th time	8	.024 in.
5th time	16	.048 in.
6th time	32	.096 in.
7th time	64	.192 in.
8th time	128	.384 in.
32nd time	2147483648	6442450.9 in.

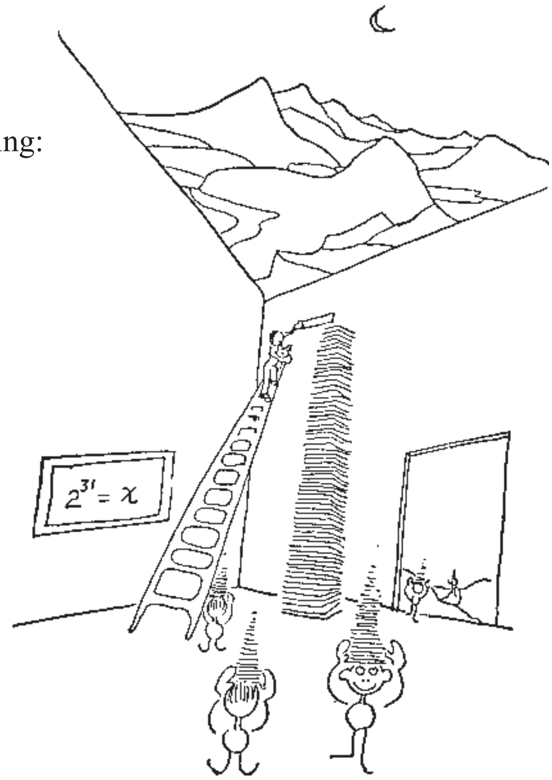
If you have the patience to carry on you’ll find that after the 32nd time the number of papers is 2147483648 and you get 6442450.9 in.

In other words, the final pile of papers is 6,442,451 in. thick.

To change this to feet, we must divide it by 12, obtaining: 536,871 feet.

Or perhaps you would like the answer in miles!
In that case, divide now by 5280, since, as you know, there are 5280 feet in 1 mile.
Thus we get: nearly 102 miles!
Remember that 1 mile is about 20 city blocks.
Now imagine a pile of papers over 100 miles in height!

Are you surprised again?
Did you get your answer by a "hunch"?
Or did you try to do it experimentally by actually piling the papers up?
Or did you calculate it as we did?



(Note: These measures use the term inches, feet, mile etc. and it is not expected that VTs actually know these relationships. This example could be modified to use whatever terms your VTs find familiar. The idea is only to appreciate the way numbers grow).

Source: The Education of T.C. MITS *(The Common Man In The Street)',* by L.R. and H.C, Lieber, W.W. Norton Co. *(1941).*

Section 3. Number Games

3.1 Tambola

Give rectangular sheets of papers to learners. Let them construct a 4 x 5 table as shown below.

Let them write any two numbers between 1 and 10, (both inclusive) in any two of the twenty squares. Again they may write any two numbers between 11 and 20, 21 -30, 31-40, 41-50................ 91 -100.

From a well shuffled pack of cards numbering 1 to 100, the Instructor shows a card. If the number on that card is any where in 20 squares, the learner can score it. This process is continued till all the cards are used.

Those who have completed any row first win that row. Similarly winners of the other rows may be determined.

72	12	78	1	82
~~67~~	~~24~~	~~25~~	~~27~~	~~55~~
48	8	57	35	97
47	64	14	83	94

Here the learner wins the second row.

3.2 Touching the Number yon Guessed :

Let there be a clock (or a diagram of it) with numbers 1 to 12 written on it. Learners may think of any one number between 1 to 12, both inclusive. The player touches the numbers on the clock at random. For every touch the learner must mentally add 1 to the number he had thought. As the learner counts 20 the player will touch the number the learner had thought.

Method:- For the first seven touches the player can choose any random number. For the eighth one, the player must make it a point to touch 12. Then go on to touch 11, 10, 9 etc........... in a decreasing sequence.

Section 4. A Date with Numbers

The Volunteer Teacher (VT) could encourage learners to talk about their 'date of birth'. How many can identify the year/month when they were born?

How many can »y when their children were born?

Does anybody have a' birth certificate'? Or the date of birth mentioned in a school certificate? When are such certificates asked for?

How can we write the date in numbers? Where the day, the month and the year are written only in numbers. What is the significance of 15-08-1947?

The VT can explain how they could write that particular date in numbers. Does anyone remember what happened on the day? After 50 years of that special day we have had a lot of talk recently. Do they see any difference in their lives or their village in these 50 years.

What would be the year after 20 years? Do they think their lives would be different then? In what way?

Expiry Date: (on medicines)

Mahendra bought some medicines from the chemist's shop and looked for the 'date of expiry' written on it. He noticed that the date was long over and the medicine was not safe for consumption. He went back and exchanged it for a fresh sample. Let's see how he had figured this out.

He had read two dates written on the medicine cover. Even though they were written in a small size these numbers were important.

Date mfd. 2-6-1994
Exp.date 2-6-1996

Let us look only at the years. The number above shows 1994. The medicine was made in this year. The second date shows the year 1996. The medicine expired in this year. Thus the medicine should now not be used. The shopkeeper must not sell such old expired stocks of medicine.

Section 5. Tricks with Numbers

5.1 Out of Seven Digits

Write the seven digits from 1 to 7 one after the other:
1 2 3 4 5 6 7.

It's easy to connect them by the plus and minus signs to obtain 40, e.g.
12 + 34 - 5 + 6 -7 = 40.

Try and find another combination of these digits that would yield 55.
There are three solutions:

123 + 4 -5 - 67 = 55;
1- 2 -3 - 4 + 56 + 7 = 55;
12 - 3 + 45 - 6 + 7 = 55.

5.2 Nine Digits

Now write out the nine digits: 1 2 3 4 5 6 7 8 9.

You can as above arrive at 100 by inserting a plus or minus six times and get 100 thus:
12 + 3 - 4 + 5 + 67 + 8 + 9 = 100.

If you want to use only four plus or minus signs, you proceed thus:
123 + 4 - 5 + 67 - 89 = 100.

Now try and obtain 100 using only three plus or minus signs. It's much more difficult but possible.

123 - 45 - 67 + 89 = 100

This is the only solution. It's impossible to arrive at the same result by using the plus and minus signs less than three times.

5.3 Unity

Obtain unity using all ten digits.

Represent unity as the sum of two fractions:
148/296 + 35/70 = 1

Those knowing more advanced mathematics may also give another answer:
$123,456,789^{0}$, $234{,}567^{9-8-1}$, etc., since any number to the zeroth power is unity.

5.4 With Five Two's

We only have five two's and all the basic mathematical operation signs at our disposal. Use them to obtain the following numbers: 15, 11

22/2 + (2 x 2) = 15
22/2 + 2 + 2 = 15
$(2x2)^2$-2/2=15
$(2+2)^2$-2/2=15

And 11 as:
22/2 + 2-2 = 11.

5.5 Once More with Five Two's

Is it possible to obtain 28 using five two's?
22 + 2 + 2 + 2 = 28

5.6 With Four Two's

Use four two's to arrive at 111. Is that possible?
222/2 = 111.

5.7 With Five Three's

To be sure, with the help of five three's and the mathematical operation signs we can represent 100 as follows:
(33 x 3 + 3)/3 = 100.

But can you write 10 with five three's?
The solution is: 33/3 - 3/3 = 10.

It's worth mentioning that the problem would have had exactly the same solution if we had to express 10 with five ones, five fours, five sevens, five nines, or, in general, with any five identical digits. In fact:

11/1 - 1/1 = 22/2 - 2/2 = 44/4 - 4/4 = 99/9 - 9/9, etc.

Also, there are other solutions to the problem:

(3 x 3 x 3 + 3)/3 = 10
$3^3/3 + 3/3 = 10$

5.8 The Number 37

Repeat the above problem to obtain 37.
There are two solutions: 33 + 3 + 3/3 = 37;
333 / (3x3) = 37

5.9 In Four Ways

Represent 100 in four various ways with five identical digits.
We can use one's, three's and (most conveniently) five's:
111-11 = 100;
33 x 3 + 3/3 = 100;
5 x 5 x 5-5 x 5 = 100;
(5 + 5 + 5+5) x 5 = 100.

5.10 With Four Three's

The number 12 can be very easily expressed with four three's:
12 = 3 + 3 + 3 + 3

It's more of a problem to obtain 15 and 18 using four three's:
15 = (3 x 3) + (3 + 3);
18 = (3 x 3) + (3 x 3).

And if you were required to arrive at 5 in the same way, you might not be very quick
5 = (3+3/3) + 3

Now think of the ways to get the numbers 1, 2, 3, 4, 6, 7, 8, 9, 10.

$1 = 33/33$ (these are also other ways);

$2 = 3/3 + 3/3$;

$3 = (3+3+3) / 3$

$4 = (3 \times 3 + 3) / 3$

$6 = (3 + 3) \times 3 / 3$.

We've given the solutions through six only. Work out the remaining ones for yourselves. The above solutions, too, may be represented with other combinations of three's.

5.11 With Four Four's

If you have done die previous problem and want some more in me same vein, try to arrive it all me numbers from 1 to 10 with four's. This is no more difficult than getting me same numbers with the threes.

$1 = 44 / 44$ or $(4 + 4) / (4 +4)$ or$4 \times 4 / 4 \times 4$ etc.

$2 = 4/4 + 4/4$, or$(4 \times 4) / (4 + 4)$

$4 = 4 + 4 \times (4 - 4)$;

$5 = [(4 \times 4) + 4] / 4$

$6 = (4 + 4) / 4 + 4$;

$7 = 4 + 4 - 4/4$, or $44 / 4-4$;

$8 = 4 + 4 + 4 - 4$, or $4 \times 4 – 4 - 4$;

$9 = 4 + 4 + 4/4$;

$10 = (44 – 4) / 4$

5.12 With Four Five's

Obtain 16 using four five's.
There is only one way: $55/5 + 5 = 16$

5.13 With Five Nine's

Can you provide at least two ways of getting 10 with the help of five nine's?
The two ways are as follows:

$9 + 99/99 =10$
$99/9 - 9/9 = 10$.

Those knowing more mathematics may add several other solutions, e.g.
$(9 + 9/9)^{9/9} =10$, or $9+ 99^{9-9} = 10$.

5.14 Twenty-Four

It's very easy to obtain 24 with three eights: 8 + 8 + 8. Could you do this using other sets of three identical digits? The problem has two solutions.

22 + 2 = 24; 3^3 - 3 = 24.

5.15 Thirty

The number 30 can easily be expressed with three five's: 5 x 5 + 5. It's more difficult to do this with other sets of identical digits. Try it; you may be able to find several solutions.
The three solutions are

6 x 6 - 6 = 30; 3^3 + 3 = 30; 33 - 3 = 30.

5.16 One Thousand

Could you obtain 1000 with the aid of eight identical digits?
888 + 88 + 8 + 8 + 8 = 1000.

5.17 Get Twenty

The following are three numbers written one below the other:
111
777
.999
Try and cross out six digits so that the sum of the remaining numbers be 20.

The crossed out digits are replaced by zeros:
011
000
009
because 11+9 = 20.

5.18 Add and Multiply

Which two numbers, when added up give 1 more than when multiplied together?

There are many such numbers, e.g.
3 x 1 = 3; 3 + 1=4;
or 10 x1 = 10; 10 + 1 = 11.

In general, any pair of integers of which one is unity will work.
This is because adding one increases a number but multiplying by one does not change it.

5.19 The Same

Which two numbers give the same result when multiplied together as when the two are added?
The numbers are 2 and 2. There are no other integers.

5.20 Three Numbers

Which three numbers give the same result when multiplied together as when they are added up?
Multiplying 1, 2, and 3 gives the same as adding them up:
1+2 + 3 = 6, 1 x 2 x 3 = 6.

5.21 Multiplication and Division

Which two integers yield the same result whether the larger of them is divided by the other or they are multiplied together?

There are many correct number pairs. Clearly, one of them is the number 1.

2 / 1 = 2, 2 x 1 = 2.

7 / 1 = 7, 7 x 1 = 7,

43 / l = 43, 43 x 1 = 43.

5.22 The Two-Digit Number (Slightly more advanced)

There is a two-digit number such that if it is divided by the sum of its digits the answer is also the sum of the digits. Find the number.

The number we seek should clearly be a square. As among the two-digit numbers there are only six squares, then by trial-and-error method we readily find the unique solution, namely 81:

81/ (8 +1) = 8+1

5.23 Ten Times More

The numbers 12 and 60 have a fascinating property: if we multiply them together, we get exactly 10 rimes more than if we add them up:

12 x 60 = 720, 12 + 60 = 72.

Try and find another pair like this. Maybe you can find several pairs with the same property.

The following are the four other pairs of such numbers: 11 and 110; 14 and 35; 15 and 30; 20 and 20.

In fact,

11 x 110 = 1.210;11 + 110=121,

14 x 35 = 490; 14 + 35=49;

15 x 30 = 450; 15 + 30 = 45;

20 x 20 = 400; 20 + 20 = 40.

The problem has no other solutions. Searching for the solutions by trial and error is tiresome and a knowledge of the ABC of algebra would make the process easier and enable us not only to find all the solutions, but also to make sure that the problem doesn't have more than five solutions.

5.24 Two Digits

What is the smallest number that you could write with two numbers? Many may believe that the number is 10. No it is 1, expressed as follows:

1 / 1, 2 / 2, 3 / 3, 4 / 4, etc., up to 9 / 9.

Those who know some more mathematics may add to these answers a number of others:

1°, 2°, 3°, 4°, etc., up to 9°

because any number to the zero power is unity.

5.25 The Largest Number

What is the largest number that you can write with four ones? Not as simple as you might think!

The commonest answer is, 1,111. But the number is far from being the largest; 11^{11} is much more, 250,000,000 times more.

5.26 Strange Multiplication Cases

(You need more patience than maths!)

Consider the following case of multiplication of two numbers:
48 x 159 = 7,632.

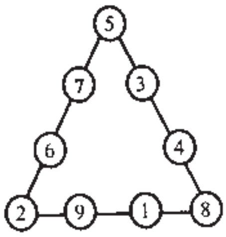
It's remarkable in that each of the nine digits is involved once here. Can you think of any other examples? If so, how many of them are there?

The patient reader can find the following nine cases where the multiplication calculations meet the question's demands. They are:

12 x 483 = 5,796,	48 x 159 = 7,632,
42 x 138 = 5,796,	28 x 157 = 4,396,
18 x 297 = 5,346,	4 x 1738 = 6,952,
27 x 198 = 5,346,	4 x 1963 = 7,852.
39 x 186 = 7,254	

5.27 Triangle of Figures

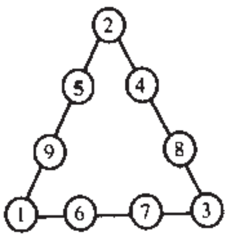
Within the circles of the triangle arrange all the nine digits so that the sum of the digits on each side be 20.



The figures in the middle of each line can be interchanged to obtain further solutions.

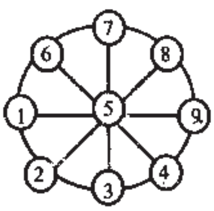
5.27 Another Triangle

Repeat the previous problem so that each side adds up to 17.
Again, the solution is given. Also, the figures in the middle of each line can be interchanged to obtain further solutions.



5.28 Wheel of Figures

The digits from 1 to 9 should be so arranged in the circles of the wheel that one digit is at the centre and the others elsewhere about the wheel so that the three figures in each line add up to 15.



(Adapted from : *Fun With Maths and Physics,* **Y.L.** Perelman, Mir Publishers, 1984)

Section 6. Number Patterns

Pattern discernment is a fruitful component of mathematical reasoning. A unit of work dealing with patterns involves students in explaining number relationships. Sequences of odd and even numbers, squares, the binary sequence, are only some of the topics that can be used to show the importance of patterns.

As an example, explore die sequence of odd numbers: 1,3,5,7,9,............

Consider the sum of the first ten (or twenty, or hundred) odd numbers. Rewrite the number in reverse order and add.

1	3	5	7	9	11	13	15	17	19
19 +	17 +	15 +	13 +	11 +	9 +	7 +	5 +	3 +	1
20	20	20	20	20	20	20	20	20	20

Notice that each vertical sum is 20. How many such sums are there? What is the sum of the first ten odd numbers? Since the first ten odd numbers have been added twice, the sum of the first ten odd numbers is
½ x10 x 20 or 100.

So far so good! But how can we use this to find a general rule for calculating the sum of as many odd numbers as we choose?

The proposed problem lends itself to several problem-solving techniques that assist in its solution. Instead of finding the sum of the first hundred, find the sum of the first four, or five. That is, show learners how to start with related problems that are easier to solve. Show them how to record data systematically so that they can look for a pattern. Is there a pattern to the sum?

ODD NUMBERS

Numbers (n)	Sum
2	1 + 3 = 4
3	1 + 3 + 5 = 9
4	1 + 3 + 5 + 7 =16

Help them experience the “Aha!” feeling.

$4=2^2$, $9 = 3^3$and $100 = 10^2$

Additional discussion of the sequence of square numbers leads to the generalization: the sum of the first n odd numbers is n^2 or n x n.

A geometric model of the numbers provides the learner with a concrete visual approach that leads to or verifies the above generalization.

1 $1+3 = 2^2$ or 2 x 2 $1+3 + 5 = 3^2$ or 3 x 3

The preceding activity may arouse students’ interest in other number sums. How about adding consecutive whole numbers! ? Start with a few - say the first four:

1 +2 + 3 + 4. See if reversing the order works this time. That is,

	1 + 2 + 3 + 4
Reverse:	4 + 3 + 2 + 1
Add	5 + 5 + 5 + 5 = 4 x 5

So, 1 + 2 + 3 + 4 = (1 / 2) x 4 x 5 or 10

There are four numbers to be added - four vertical sums, each one more than the largest number, 4. This yields 4 x 5 or 20. But, since this is twice the required sum, the answer is 1/2 x 20 or 10. Try another! To calculate the sum of the first five numbers, write:

	1 + 2 + 3 + 4 + 5
Reverse:	1 + 4 + 3 + 2 + 1
Add:	6 + 6 + 6 + 6 + 6 = 5 x 6

Therefore, 1 + 2 + 3 + 4 + 5 = ½ x 5 x 6 or 15

Summarize these explorations and look for a pattern. The VTs may find this interesting, especially if they have had to memorise some such formulae without understanding me elegant pattern behind them!

WHOLE NUMBERS

Numbers (n)	Sum
2	1 + 2 = ½ x 2 x 3 or 3
3	1 + 2 + 3 = ½ x 3 x 4 or 6
4	1 + 2 + 3 + 4 = ½ x 4 x 5 or 10
5	1 + 2 + 3 + 4 + 5 = ½ x 5 x 6 or 15

Is the sum of the first six numbers 1/2 x 6 x 7? Try it and see! Now generalize. The sum of the first n consecutive numbers is (1/2) n (n +1). The sum of the first twenty numbers is 1/2 x 20 x 21; the sum of the first hundred numbers is 1 /2 x 100 x 101; and the sum of the first thirty five numbers is (1/2) 35 x 36.

Section 7

Number Stories

7.1 Mollakka's Horse

Once upon a time their lived a businessman. He had three sons. None of them was interested in his business. The transactions were carried out by his manager. Accidentally one day he fell ill. In his last days he prepared a will. In it he mentioned that half of his property should go to the first son. Half of the remaining should go to the second; and half of the remaining should go to the third. After his death they realised that only seven horses are left as their father's property.

In order to divide the property as in the will they would have to cut the horses. So they were in a dilemma. Then a wise man called 'Mollakka' came to help them. He first gave his horse to them; as a gift. Then the total property became 8 horses. As mentioned in the will, the first son got half of the total, i.e. 4 horses; the second son got half of the remaining 4, i.e. 2 horses and the third son got half of the remaining 2, i.e. 1 horse. All together there were 4 + 2 + 1=7 horses. Mollakka returned home riding his own horse.

7.2 The Way the Maharaja Counted his Horses

A non-literate Maharaja has some horses. He does not know the exact number. He knows only one thing. There are 9 horses in each line of the stable where they are kept. Usually they are arranged like:

3	3	3
3	3	3
3	3	3

How many are they in all?

One day a traveler came that way with 4 horses. **He** wanted to keep the horses in the royal stable for one night. But the man in charge of the horses hesitated. The Maharaja would get angry on finding the traveler's horses tied in his stables. The traveler said that he would manage to avoid to attract the Maharaja's attention. He then arranged the horses as:

2	5	2
5		5
2	5	2

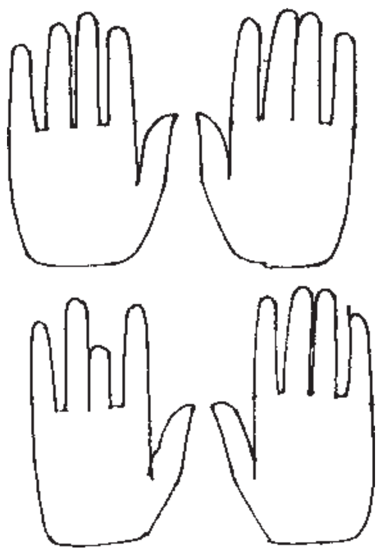
The Maharaja came on his usual round of the stables and was satisfied with everything. He did not notice the extra horses. The next morning die traveler managed to slip away quickly. The arrangement of the horses he left behind was:

4	1	4
1		1
4	1	4

The Maharaja again noticed no change. But do you? How many of the Maharaja's horses did the traveler take with him?

Section 8. Fun with Arithmetic

8.1 Multiplication Using Your Fingers



If you don't remember the multiplication table properly and have difficulty in multiplying by 9, then your own fingers might be of help. Place both hands on a table, your 10 fingers will be your computer.

Suppose you want to multiply 4 by 9.
Your *fourth* finger gives the answer: on the left of it there are three fingers, on the right, six. So you read: 36, hence 4 x 9 = 36.

Further examples: how much is 7 x 9?
Your *seventh* finger has six fingers on its left and three on its right. The answer: 63.

What is 9 x 9? On the left of the ninth finger there are eight fingers, on the right, one. The answer is 81.

This living computer will remind you, for example, that 6 x 9 is 54, not 56!

8.2 Cats and Mats

Once some cats
found some mats.
But if each mat
had but one cat
there'd be a cat
without a mat.
Should each mat
now have two cats
there'd be a mat
without a cat.
How many cats
and how many mats?

Answer :-This problem is solved in this way. Ask the question: How many more cats would be needed to occupy all the places on the mats the second time than, to get the situation we had the first time? We can easily figure that out: in the first case one cat was left without a place, whilst in the second case all the cats were seated and there were even places for two more. Hence for all the mats to have been occupied in the second case there should have been 1 + 2;i,e. three, more cats than there were in the first case. But then each mat would have one more cat. Clearly there were three mats all in all. Now we seat one cat on each mat and add one more to obtain the number of cats, four.

Thus, the answer is four cats and three mats.

8.3 Sisters and Brothers

I have an equal number of sisters and brothers. But my sister has two times more brothers than sisters. How many are we?

Answer:-Seven: four brothers and three sisters. Each brother has three brothers and three sisters; each sister has four brothers and two sisters.

8.4 How Many Children?

11 have six sons. Each son has a sister. How many children have I?

Answer :- Seven: six sons and one daughter. (The common answer is twelve, but each son would then have six sisters, not one.)

8.5 Fishy

Two fathers and two sons ate a hearty dinner with three fried fish, each having a whole fish. How do you account for it?

Answer :- The situation is very simple. Eating together were three, not four people: a grandfather, his son, and his grandson. The grandfather and his son are fathers, and the son and grandson are sons.

8.6 Who is Older?

In two years my boy will be twice as old as he was two years ago. And my girl in three years will be three times as old as she was three years ago.

Who is older, my boy or my girl.

Answer :- Neither is: They are twins and each of them at the time is six years old.
The calculation is simple: two years hence the boy will be four years older than he was two years ago, and twice as old as he was then. Hence he was four years old two years ago. Accordingly, now he is 4 + 2 = 6 years old.

The girl's age is the same.

8.7 Snail

A snail was climbing up a 15 metre tree. Each day it climbed 5 metres, but each night as it slept it slid back down 4 metres.
How many days did ii take the snail to reach the summit?

Answer:-In 11 days. During the first 10 days the snail had crawled up 10 metres, 1 metre a day. The next one day it climbed the remaining 5 metres, i.e. it reached the summit. (The common answer is 15 days.)

8.8 Two Schoolboys

A schoolboy said to his mate, "Give me an apple, and I'll have twice as many as you."
"That would be unfair" replied the mate, "You give me one then we'll be even."
How many apples had each initially?

Answer :- Transferring an apple balances out the number of apples, thus suggesting that one had two apples more than the other. If we subtract one apple from the smaller number and add it to the larger number, then the difference will increase by two and become four. We know that then the larger number will be equal to double the smaller one. Accordingly, the smaller number is 4 and the larger 8.

Before the transfer one schoolboy had 8 – 1 = 7 apples, and the other 4 + 1 = 5 apples.

Let's check whether or not the numbers become equal if we subtract an apple from the larger and add it to the smaller:

$7 - 1 = 6; \quad 5 + 1 = 6.$

Thus one schoolboy has 7 apples and the other 5 apples.

8.9 Spiders and Beetles

A boy collects spiders and beetles in a box and he now has 8 insects in all. There are 54 legs in the box in total. How many spiders and how many beetles are there?

Answer :- **To** tackle the problem we should first of all remember how many legs beetles have and how many spiders have. In fact, the numbers are six and eight, respectively. With this in view we suppose that the box only contains beetles. Their legs would then add up to 6 x 8 = 48, six fewer than given in the problem. Let's now try and replace one beetle with one spider. This will increase the number of legs by two because the spider has two more legs.

Clearly three such replacements will bring the total number of legs in the box up to the desired 54. But in that case there will only be five beetles, the rest being spiders.
The box thus contained five beetles and three spiders.

Let's check: five beetles give 30 legs, the three spiders 24, the total being 30 + 24 = 54, as required.

The problem can also be solved in another way. We may start off assuming that the box only contains spiders, eight of them. The number of legs more than what was stated. Replacing one spider with one beetle reduces the number of legs by two. We'll have to make five such substitutions in order to arrive at 54. Put another way, we'll retain only three spiders, with the rest being replaced by beetles.

8.10 Reversible Numbers

A 'PALINDROME' is usually defined as a word, sentence, or set of sentences that spell the same backward as forward. The term is also applied to integers that are unchanged when they are reversed. Both types of palindromes have long interested those who amuse themselves with number and word play. Let's take an example. Start with any positive integer. Reverse it and add the two numbers. This procedure is repeated with the sum to obtain a second sum, and the process continues until a 'palindromic sum' is obtained. A palindrome always results after a finite number of additions. For example, 68 generates a palindrome in three steps:

68
+ 86
154
+ 451
605
+ 506
1,111

For all two-digit numbers if the sum of their digits is less than 10, the first step gives a two-digit palindrome. If their digits add to 10, 11, 12, 13, 14, 15, 16, or 18, a palindrome results after 2, 1, 2, 2, 3, 4, 6, 6 steps respectively. You may check this yourself, and also entertain yourself in the process!

8.11. Magic Squares

Since time immemorial people have amused themselves by constructing magic squares- His problem consists in arranging successive numbers (beginning with 1) aver the cells of a divided square so that the numbers in all the lines, columns and diagonals add up to the same number.

The smallest magic square has nine cells. It can easily be shown by trials that a four-cell magic square is impossible- The following is an example of a 9 - cell magic square:

4	3	8
9	5	1
2	7	6

In this square we might add up either 4 + 3 + 8, or 2 + 7 + 6, or 3 + 5 + 7, or 4 + 5 + 6, or any other line of three numbers, the result is always 15. The result could be envisaged beforehand, without constructing the square as such: the three lines of the square should contain all the numbers and they add up to 1+2+3+4+5+6+7+8+9=45.

On the other hand, this sum must clearly be equal to thrice the sum of a single line. Hence for each line 45 / 3=15.

Using the same argument we can determine in advance the sum of the numbers in a line or column of any magic square consisting of an arbitrary number of cells. We only have to divide the sum of all its numbers by the number of its lines.

Zero Sum Magic Square
To construct your own magic squares, it is useful to remember the basic skeleton - the Zero Sum Magic Square, starting with a 0 at the centre.

1	2	- 3
-4	0	4
3	-2	- 1

Now suppose we want to make a magic square with 5 at the centre, just add 5 to all me numbers. See what you get.

Magic squares have fascinated ancient mathematicians both in China and India. In the 16th century Sundara Suri worked out the basic 4 x 4 Zero Sum Magic Square. You want to try?

8.12 Fifty Million People Can Be Wrong!

Let us begin with a very simple question:
suppose you had the choice of the following two jobs:

Job 1: Starting with an annual salary of Rs. 1000, and a Rs. 200 increase every year.

Job 2: starting with a semi-annual salary of Rs. 500, and an increase of Rs. 50 every 6 months.

In all other respects, the two jobs are exactly alike.
Which is the better offer (after the first year)? Think carefully and decide on your answer
Did you say Job 1 is better?
And did you reason as follows?
Since Job 2 has an increase of Rs. 50 every 6 months, it must have an annual increase of Rs. 100 and therefore it is not as good as Job 1 which has an annual increase of Rs. 200,
Well, you are wrong!
For, examine carefully the earnings written oat below:

		1st half of year	2nd half of year	Total for the year
1st year	Jobl	Rs.500	Rs.500	Rs. 1000
	Job2	Rs.500	Rs.550	Rs. 1050
2nd year	Job1	Rs.600	Rs.600	Rs. 1200
	Job2	Rs.600	Rs.650	Rs. 1250
3rd year	Jobl	Rs.700	Rs. 700	Rs. 1400
	Job2	Rs.700	Rs. 750	Rs. 1450
4th year	Jobl	Rs.800	Rs.800	Rs. 1600
	Job2	Rs.800	Rs.850	Rs. 1650

Note that:

(1) Job 1 pays Rs. 200 more each year than it did the previous year.

(2) Job 2 pays Rs. 50 more every half-year than it did during the previous half-year.

All this is in accordance with the promises originally made, and yet Job 2 brings in Rs. 50 more every year than Job 1 does. And you can easily see that this will continue to be true no matter what number of years is considered. You are probably surprised. But don't be discouraged, for you are in plenty of good company. Try it on your friends, and you will find that, unless they have heard it before, they will probably make the same mistake that you made. Fifty million people CAN be wrong! And this is entirely normal.

Excerpts From the book *'The Education of T.C. MITS (The Common Man in the Street)'* by L,R. Lieber, W.W. Norton Co. (1942)

Section 9. Counting Different Things All at Once

Can you count?

The question might seem to be even insulting for a person more than three years old. Who can't? To utter the words "one", "two", "three" etc., in succession doesn't take much genius. And still I'm sure that you're not always equal to (his seemingly simple task. Everything depends on what is to be counted. It's no problem to count, say, the nails, in a box. But suppose the box contains screws as well as nails. It's required to find out how many of each there are. How could you go about it? Would you separate the heap into nails and screws and then count them?

This sort of a problem comes up for a washerman (dhobi) when he has to count the washing for laundry. He first sorts the washing out: shirts go to one heap, towels to another, etc. And it's only after he had done this tedious job that he begins to count the items in each heap.

That's what may be called not knowing how to count! This way of handling dissimilar objects *is* utterly inconvenient, labour consuming and occasionally even completely impossible. It's all well and good if you have to count nails or washing: they are fairly easily sorted out into separate heaps. But try to place yourself into a forester's shoes who wants to count all the teak, neem, palm, and banana trees in the same hectare. He cannof sort out all the trees according to their species. Well, should you first count all the teak, then all the neem, then all the palm and then all the bananas. Would you go all round the whole area four times?

Couldn't the job be done in a simpler way, perhaps by a single tour of the area? Yes, there is such a way and it has been used since time, immemorial by foresters. We'll illustrate its principle essence referring to our nails and screws.

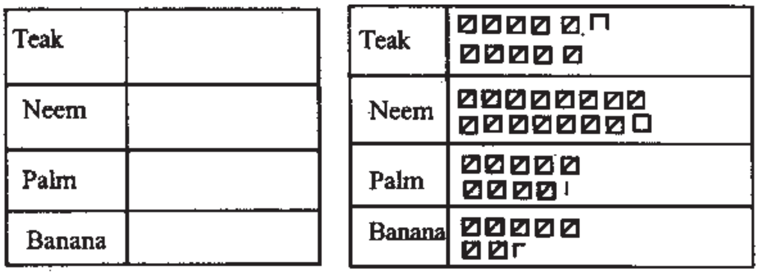
To count the nails and screws at one go, without sorting them out, get a pencil and a sheet of paper marked out as shown below:

Nails Screws

Now begin counting. Take out of the box whatever comes first. If it's a nail you make a dash in "Nails", if it's a screw, mark a dash in "Screws", Take out a second piece and repeat the procedure, then a third, a fourth, etc., until the box is finished. In the end, in the "Nails" column you'll have as many dashes as there are nails in the box, and in the "Screws" column as many dashes as there are screws. It only remains to count up the dashes.

We could simplify the counting procedure. To do so we should not just dispose our dashes one under another, but group them as shown with five dashes in each group.

It's easy to count the dashes thus arranged: you see at once that here we have three complete tens, one five plus three dashes,
i.e. 30 + 5 + 3 = 38.



You'll end up with about what is shown in figure. It's a straightforward exercise here to work out the totals:

Teak	53
Neem	79
Palm	46
Banana	37

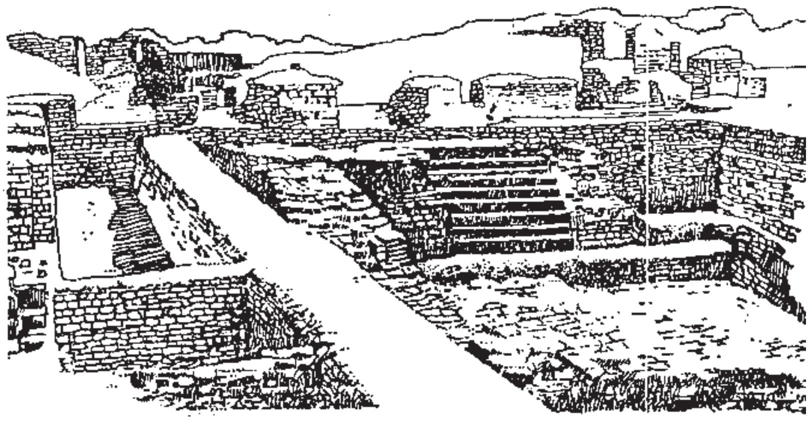
The same procedure is used by a medical worker who counts under the microscope red and white blood corpuscles in a blood specimen.

Should you need to count the plants of various species in a meadow you'll now know how to handle the job and do it in the shortest time possible. First write down the names of the plants found and allot a line to each, leaving several lines for other plants you may come across.
(Adapled from *'Fun With Maths and Physics'* Y.L. Perehman, Mir Publishers, 1984)

Section 10. The Story of Zero

Do you know where zero was invented? India. It was then called 'shoonya'. As it travelled to distant lands, it acquired different names until it was widely accepted as 'zero'. It has had a fascinating story.

Indians were adept at mathematics from the days when the cities of Mohenjo-daro and Harappa prospered, almost 5000 years back. The uniform size of their bricks, accurate weights and measures and systematic town planning throw much light on the mathematical mind of the inhabitants. In ancient India, mathematics, known as 'Ganita', the science of calculation, was held in high esteem. The numbers, namely 1, 2, 3......9,
as we know them today, were called 'Ankas' - marks or divisions. It is, however, not known when these number symbols were invented. But it can be easily guessed that they were invented before the symbol for zero came into use.

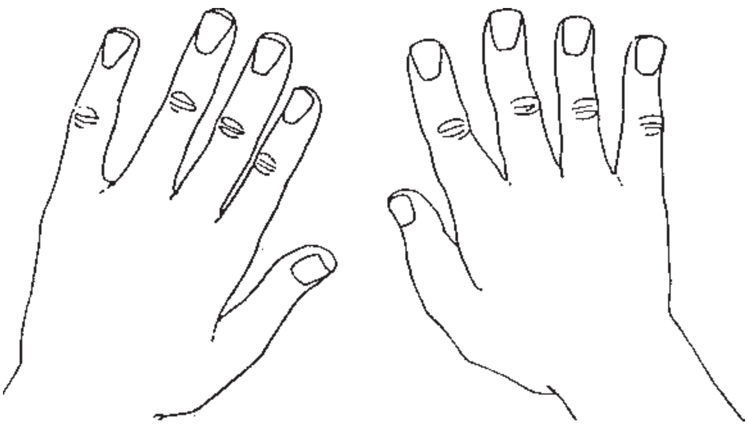


The counting of numbers in tens, as we do today, is known as the decimal system, and the numbers as decimal numbers. The stone-pillars erected during the reign of Emperor Asoka (273 B.C. - 232 B.C.) indicate the decimal system and number symbols. However, it is thought that in those times the symbols were used only for small numbers. Very big numbers were written in words. For instance, 1,000 was called 'sahasra', 10,000 'aayuta', 100,000 'laksha', 10,000,000 'koti' and soon. Numbers were named after things seen or used in daily life. For instance, moon or earth represented one, eyes or hands or twins represented two, and so on.

It is said that thinking up big numbers and naming them was the favourite pastime of ancient Indian mathematicians. While making up big numbers, they would employ the fingers of their hands. As there are five fingers to each hand, making a total often, their counting system had ten numbers and multiples often. This system of measuring numbers in terms often, whether multiplying or dividing, is known as the decimal system. For instance,

1/2 is taken as 1 x 10 = 5/10 = 0.5   where the dot is the decimal point.
                2 x 10

Similarly, 120 = 12 x 10 and so on. This is how the decimal system began in India.



Likewise, the 'position value notation' came into being when the ancient mathematicians wrote down a mass of numbers in words. In due course, this system was adopted in writing the number symbols. To explain position value notation, take the number 7,456. It can be broken down into multiples often thus:

7,456 = 7 x 10 x 10 x10 + 4 x 10 x 10 + 5 x 10 + 6
= 7 x 1,000 + 4 x 100 + 5 x 10 + 6
= 7,000 + 400 + 50 + 6

In other words, it is 7's position in the number that makes it seven thousand. Likewise, 4'sposition makes it four hundred, and so on. It is, therefore, the position of a digit in a number that determines its value.

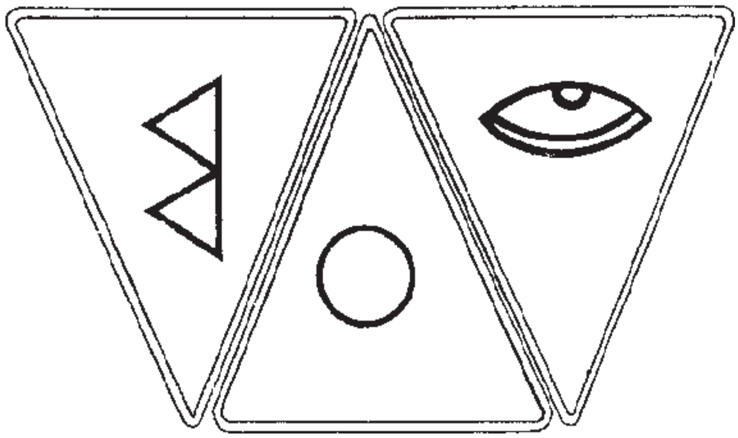
The earliest reference to position value notation is found in Agni Purana, an ancient Indian text written about a century after the birth of Jesus Christ. Ancient Indian mathematicians had, therefore, the skill to write numbers as big as 10^{18} (ten multiplied by 10, eighteen times), while ancient Greek and Roman mathematicians could count only up to 10^4 or 10 x 10 x10 x 10, and 10^3 or 10 x 10 x 10 respectively. The number system of the Romans (which used letters like M, C, L, X, I, etc.) did not allow them to count beyond a particular number and to invent the position value notation.

However, in ancient India only the learned had the privilege of counting numbers and performing calculations. Nevertheless, mathematics continued to be a respected subject, a tradition that continued even when Buddhism and Jainism spread in the land. Through Buddhism, Indian numbers spread to China and Japan. Some Hindu merchants settled down in the countries of the Far East and introduced Indian numbers there.

It was, however, not until zero was invented that mathematics prospered in India. The decimal numbers and position value notation did not click till zero was added. The position value notation makes no sense without zero because it is zero that makes 26, for instance, different from 206 or 20,006. It is the position of zero in the number that adds a new meaning to it. Calculations also became simpler and faster.

Nothing is known about who invented zero, where or when. It is claimed that Indians knew about it even before the Christian era. The ancient seer Pingala and the political philosopher and statesman Kautilya have mentioned it several times in their works.

In those ancient times, zero was represented as a circle with a dot at its centre. In Sanskrit, the language then popular in India, it was called ‘shoonya’ - void or emptiness. In those days, when Sanskrit was spoken all over the country, zero had several names like ‘kha’, ‘gagana’ ‘akasha’, ‘nabha’, ‘ananta’ which denote sky, void or endless.



The Mayas of Central America had also invented zero but they did not know the principle of numbers, as Indians did, to enable them to develop mathematics. Initially, zero was invented to signify ‘nothing’ in the place of a number. If somebody had seven mangoes and all seven were eaten, ‘nothing’ was left. The Mayas of Central America had invented zero m this sense only. It took the ingenuity of Indian mathematicians lo understand the significance of zero and accept it as an additional “number”. Their subtle mind gave the status of a number for ‘nothing’. The eminent Multan-born Indian mathematician Brahmagupta (598 A.D. - 660 A.D.) went on to give the rules of operation of zero in his treatise Brahmasphutasiddhanta as though zero were any other number. Today, his rules may sound trivial, but imagine their significance when zero was ‘nothing’ in the rest of the world. Brahmagupta claimed.

A + 0 = A, where A is any quantity
A - 0 = A
A x 0 = 0
A / 0 = 0   (this was not correct!)

In the case of division of a quantity by zero Brahmagupta faltered. The division of a quantity by zero is ‘infinity’ and not zero.

This mistake was corrected five hundred years later by another greal Indian mathematician Bhaskara, a native of Bijapur, Kamataka. In his famous book Leelavati, Bhaskara claimed that the division of a quantity by zero is an infinite quantity “which does not change when worlds are created or destroyed.”

Meanwhile, the creation of zero led Indian mathematicians to think of numbers less than zero itself. So came into circulation the negative numbers, namely, -1, -2, -3. From sixth century to tenth century for about 400 years, India became the centre of mathematics in the world. Naturally, at that time, the fame of mathematics in India had spread far and wide. Its applications in astronomy and other subjects had also been made.

Even earlier than the eleventh century, important works of Indian mathematicians had already been passed on to the Arab world. With the rise of the Arab civilization, trade exchanges among Arabs, Greeks and Indians had begun. Sometimes, scholars also accompanied the caravans of traders and merchants to visit new lands and gain knowledge. Under the Arab ruler of Baghdad, a number of emissaries and scholars were sent to Sind (now in Pakistan) in the eighth century to learn astronomy, mathematics and medical science in India. Not only were a number of great Indian texts on various sciences, including mathematics, brought to Baghdad for study but also translated into Arabic.

Arab mathematicians read Indian works and also began to use Indian numbers, including zero. The great Arab mathematician Al-Khowarizmi visited India and saw Indian mathematicians performing calculations with ease and speed. Back in Baghdad, he wrote his famous treatise *Hisab-al-Jabr wa-al-Muqabala* (Calculation for Integration and Equation) which caught the attention of the Arab world and made Indian numbers popular. ‘Shoonya’ became ‘al-sifr’ or ‘sifr’. One can judge the impact of his treatise from the fact that the title ‘al-Jabr’ became the ‘Algebra’ of today.

When Europeans came to know of these numbers they called them ‘Arabic numbers’, and that is how they are still known. In fact, the word ‘Shoonya’ became ‘sifr’ in Arabic, then ‘Zephirum’ in Latin. As the knowledge of numbers travelled through different countries ‘O’ was called by many local European names like ‘zenero’, ‘iziphra’, ’Zephiro’, ‘cypher’, ‘zero’ etc. The story of zero is thus a fascinating history of how Indian numbers travelled across the world and helped in the development of mathematics.

(Adapted from the book by Dilip M. Salwi, Children’s Book Trust)

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